Math.520.001 Introduction to Analysis 1 Fall 2011

Extra Problem for Section 10

1. Prove: If (a_n) and (b_n) are Cauchy sequences, so is $(a_n + b_n)$.

Optional Bonus Problem for Students Who Crave an Extra Challenge

2. Let the sequence (F_n) be defined as follows: $F_1 = 1$, $F_2 = 1$, and for any $n > 2$, F_n is the sum of the two preceding terms of the sequence. In other words, (F_n) is the Fibonacci sequence 1, 1, 2, 3, 5, 8, …

   Next define a new sequence by $a_n = F_{n+1} / F_n$. The object of this set of exercises is show that the sequence (a_n) converges and that $\lim a_n = \frac{1+\sqrt{5}}{2}$.

   a. Show that $a_{n+1} = 1 + \frac{1}{a_n}$ for all $n > 0$.

   b. Show that $a_n \geq 1$ for all n.

   c. Show that $a_n \leq 2$ for all n.

   d. Prove that the sequence (a_n) converges. This is the tricky part. As a first step, I suggest you compute the first several terms in decimal form and study the pattern of increases and decreases. This should lead you to some conjectures about the even and odd terms of the sequence.

   e. Show that $\lim a_n = \frac{1+\sqrt{5}}{2}$.