

## Math 520 Introduction to Real Analysis 1 Sample Prerequisite Proof Problems

The following are typical problems from Math 503. Students in Math 521 should be comfortable finding the answers and formulating valid proofs.

1. Proof by induction. Suppose  $a_1 = 1$  and  $a_{n+1} = \sqrt{1+a_n}$  for all  $n > 0$ . Prove that  $a_n < 2$  for all  $n \geq 0$ .

2. Proofs of set equality. If  $A$  and  $B$  are subsets of a set  $X$ , prove that

$$(X - A) \cup (X - B) = X - (A \cap B)$$

3. Indexed collections of sets. For each  $\lambda \in \Lambda$  let  $A_\lambda$  be a subset of the set  $X$ . Prove that  $\bigcup_{\lambda \in \Lambda} (X - A_\lambda) = X - \bigcap_{\lambda \in \Lambda} A_\lambda$ .

4. Properties of abstract functions. For a function  $f : A \rightarrow B$ , which, if either, of the following must hold? For each one prove it is true or give a counter example to show it is not true in general.

(a)  $f[f^{-1}(C)] \subseteq C$  for all  $C \subseteq B$ .

(b)  $f^{-1}[f(D)] \subseteq D$  for all  $D \subseteq A$

5. Quantifiers  $\forall$  and  $\exists$ . Prove:  $\forall n \in \mathbb{N} \exists x \in \mathbb{R}$  with  $0 < x < 1/n$ .