Derivative Securities

- **Definition**
  - A derivative security or derivative
  - is a financial instrument
  - whose value depends on
  - the values of other more basic underlying variables

- **Examples**
  - Options
  - Forward & Futures
  - Swaps

Terminology

- **Number of parties**
  - 2 (buyer & seller) + intermediaries (sometimes)

- **The party that has agreed to:**
  - **BUY**
    - has what is termed a *LONG* position
    - the long position gains when the price of the underlying increases
  - **SELL**
    - has what is termed a *SHORT* position
    - short gains when the underlying’s price falls
Terminology 2

• Types of Traders
  • Speculators
    » are willing to take risk based on their forecasts
    » try to exploit price movements
    – “Investors”?
    » use derivatives to gain LT (as opposed to ST) exposure
  • Hedgers
    » want to reduce risk of existing assets or liabilities
  • Arbitrageurs
    » use risk-free trading strategies
    » to exploit asset mis-pricings

Ways Derivatives are Used

• To invest or speculate
• To hedge risks
• To infer views
  • about the future direction of the market or about risk
• To lock in arbitrage profits
• To change the nature
  • of a liability
  • of an investment

Forwards: Fundamentals

• Definition
  • contract calling for delivery of a given asset
  • at a given future date, at a price agreed-upon today
  • no money changes hands today (caveat)
• Market participants & Payoffs (Who & Why?)
• Market microstructure (Where and How?)
  • OTC market
  • Possible underlying assets (including FX)
  • Forward quotes for currency contracts
Forward Terminology

• Commodity-settled contract:
  • Forward contract that requires delivery of the underlying asset against payment in full
  • Typical for currencies (settlement by exchange of bank balances at correspondent banks)

• Non-deliverable forward (NDF, also CFD):
  • Contract that requires the parties to make a cash settlement ONLY on the difference between the forward and the spot price at maturity
  • Used when the parties know they’ll be unable (or unwilling) to take delivery of the underlying asset

Forwards 2: Who Trades What?

• Market participants
  • hedgers
  » try to avoid impact of price movements
  » short hedgers: have long underlying position, go short
  » long hedgers: have short underlying position, go long
  • speculators
  » try to profit from price movements
  • traders-arbitrageurs

Forwards 3: Results from Trading

• Payoff at maturity (or expiration or delivery)
  • What matters?
    – long position vs. short position
    – hedged position vs. naked position
    » long hedge vs. short hedge
  • Regrets, anyone?
Forwards 4: Payoffs from Naked Positions

- **Definition**
  - “Naked”
    - means the forward position holder does not have a position in the underlying asset
    - i.e., neither owns nor owes the underlying asset
  - **Interpretation**
    - Speculator / Investor
    - US regulatory terminology: “Non-Commercial”

### Profit/Loss from a Long NAKED Forward Position

![Graph showing profit vs. the price of the underlying at expiration](image)

**Profit/Loss** from a Long NAKED Forward Position

- Profit
- **Price of Underlying at Expiration** ($S_T$)
- **Forward Price at Time** ($F_{t,T}$)

**How does the Long’s payoff come about?**

- **Commodity-settled forward**
  - Go long forward at time $t$ for delivery at time $T$
  - at time $T$, long receives the underlying & pay $F_{t,T}$
  - at time $T$, long resells the underlying (spot) for $S_T$
  - Long’s profit/loss is $S_T - F_{t,T}$

- **Cash-settled forward**
  - a.k.a “NDF” (Non-Deliverable Forward)
  - No delivery, only cash settlement between Long & Short
    - Long gets $\{S_T - F_{t,T}\}$ from Short (who gets $\{F_{t,T} - S_T\}$)
Profit/Loss from a Short NAKED Forward Position

How does the Short’s payoff come about?

• Commodity-settled forward
  
  Go short forward at time $t$ for delivery at time $T$
  
  ➔ at time $T$, short delivers the underlying & gets $F_{t,T}$
  
  ➔ short is “naked”, so at time $T$ short must buy the underlying (spot) for $S_T$
  
  ➔ Short’s profit/loss is $F_{t,T} - S_T$

• Cash-settled forward
  
  • No delivery (cash settlement only, between Long & Short)
    
    ➔ Short gets $(F_{t,T} - S_T)$ while Long gets $(S_T - F_{t,T})$

Forwards 5: Payoffs from Hedged Positions

• Definition
  
  • “Hedged”
    
    ➔ means the forward position holder has a position in the underlying asset
    
    ➔ i.e., either owns or owes the underlying
    
    ➔ Long the underlying ➔ hedge by going short
    
    ➔ Short the underlying ➔ hedge by going long

• Interpretation
  
    ➔ US regulatory terminology: “Commercial”
Payoff from a HEDGED Forward Position

- Hedging with “commodity-settled” contract
  - Examples: Long vs. Short hedge

- Hedging with “cash-settled” contract
  - Equivalence of hedging results
  - Advantages?
  - Examples: Long vs. Short hedge

Payoff from a (commodity-settled) HEDGED Long Position in Underlying

\[ F_{LT} \]

Cash-Inflow

Payoff independent from \( S_T \), i.e., price of Underlying at Expiration

\[ S_T \]

Payoffs from using NDF to HEDGE a Long Position in Underlying

\[ F_{LT} \]

Proceeds from spot sale of underlying

Net Cash-Inflow

\[ S_T \]

Profit from short NDF position
Forwards vs. Options

- A forward contract gives the holder the obligation to buy or sell at a certain price.
- An option contract gives the holder the right (but not the obligation) to buy or sell at a certain price.

Option Payoffs at Maturity Payoffs

- Summary: \( X \) = Strike price; \( S_T \) = Price of underlying asset at maturity.

Payoff

\[
\begin{array}{cc}
\text{Payoff} & \text{Payoff} \\
X & S_T \\
S_T & X \\
X & S_T \\
\end{array}
\]

Forwards 6: What gets traded?

- Underlying assets
  - Most important ones (in terms of volume & notional value)
    - Foreign exchange
      - Outright currency forwards and FX swaps
    - Interest rates
      - Forward Rate Agreements (FRAs)
    - Equities and Commodities
  - Relative importance
    - See semi-annual BIS figures (handout dt1920a.pdf)
Forwards 7: How does one trade?

• General logistics: same as spot
  • bid vs. ask
    » bid = price at which market maker buys from customers
    » ask = price at which market maker sells to customers

• One complication…

Currency Forward Quotes

• Forward quotes (FX)
  • outright forward vs. “swap rate” (forward premium)
    » spot 1.6275 - 1.6299 $ / 1£
    » “swap rate” 33 - 46
    » outright forward 1.6308 - 1.6345 $ / 1£

• outright forward vs. “swap rate” (forward discount)
  » spot 1.6275 - 1.6299 $ / 1£
  » “swap rate” 45 - 33
  » outright forward 1.6230 - 1.6266 $ / 1£

Currency Forward Quotes 2

• Swap rate & B-A spread
  • observation
    » subtract swap if discount, add if premium
    » why? (size of B-A spread)
  • explanations
    » risk?
    » liquidity (market depth)?
    » others?
Currency Forward Quotes 3

• Annualizing the forward premium/discount
  • Example:
    spot: $ 1.6275 / £1
    3-month outright forward: $ 1.6250 / £1
  • Swap rate
    ≈ f-s = -0.0045 $ / £1 or discount of 45 “points”
  • Percentage premium/discount
    ≈ (f-s)/s = -0.0045/1.6275 or -0.28%
  • Annualized percentage premium/discount
    ≈ (f-s)/s*4 = -0.0111 or -1.11%

Forwards 8: Misc.

• Regulation?
• Risks
  • volatility of underlying asset price
  • default
    » why?
• Solution
  • Till 2010: currencies vs. most other assets (futures)
  • Since then: impact of Dodd-Frank Act?

Links between Forward & Spot Prices

• A. Relationship to current spot (E&R Ch.6 pp.139-45)
  • Theory: (Covered) Interest Rate Parity
    » Derived through arbitrage considerations
    » (C)IRP = forward-spot parity for currencies
  • Practice: Exploiting deviations from IRP

• B. Relationship to expected future spot
  • Theory: (UN-covered) Interest Rate Parity
    » “Guesstimate” – you may lose your shirt if you bet on it
    » (U)IRP is aka “forward parity”
  • Practice: Carry trades (E&R Ch.6 pp.145-6)
A. Relationship to Current Spot

- **Forward-Spot Parity**
  - relationship between fwd price & current spot price
  - basic idea:
    - (1) borrow today
    - (2) buy an asset
    - (3) go short forward on the asset
    - the position should have zero risk & zero cost
    - hence, the rate of return should be zero

### Relationship to Current Spot 2

- **Foreign currency case**
  - (1) borrow local $S_0$ - $S_0(1+r)$
  - (2) buy 1 FX unit - $S_0 + 1 \times FX$
  - receive interest + $r^* \times FX$
  - (3) short currency forward + $F_0(1+r^*)$
  - total $\theta - S_0(1+r) + F_0(1+r^*)$

- **hence:** $\theta = F_0(1+r^*) - S_0(1+r)$ and thus $F_0 = S_0(1+r)/(1+r^*)$

### Relationship to Current Spot 3

- **Interest rate parity theorem**
  - Forward-Spot Parity for currencies
  - Adjust for timing conventions
    \[ f_{T,t} = \frac{1 + \frac{T-t}{360}}{1 + \frac{r^*}{360}} \quad \text{or} \quad f_{T,t} = \frac{1 + \frac{r}{360}}{1 + \frac{r^*}{360}} \]
  - Be careful with AS, NZS, £
    - T/365 vs. T/360
Relationship to Current Spot 4

• **Question**
  - Suppose that the 3-month interest rate in Denmark is 3.5%.
  - Meanwhile, the equivalent interest rate in England is 6.5%.
  - Both rates are annualized.
  - What should be the annualized 3-month forward discount or premium at which the Danish krone will sell against the pound?

Relationship to Current Spot 5

• **Answer**
  - the Danish krone (DKr) should sell at a premium against the pound approximately equal to the interest rate differential between the two countries
  - \( f_{ft} = \frac{(i_£ - i_{DKr}) \frac{T}{360}}{1 + i_{DKr} \frac{T}{360}} = \frac{6.5\% - 3.5\%}{1 + 3.5\%} \frac{90}{360} = 0.7435\% \)

B. Relationship to Expected Future Spot

• **Forward & expected future spot**
  - convergence property (maturity only)
  - expectation hypothesis
    - no uncertainty
    - risk neutrality
  - empirical evidence
    - short term: does not work (caveat: fancy e’trics)
    - long term: works much better (Chinn, JIMF ’06)
Relationship to Expected Future Spot 2

- Convergence of Forward to Spot

![Diagram showing convergence of forward to spot price over time]

Relationship to Expected Future Spot 3

- Forward Parity
  - speculative efficiency
    - hypothesis (NOT arbitrage)
      \[ f_{t,T} = \mathbb{E}[s_{t+T} | l_t^M] \]
    - is efficiency consistent with a risk premium?
      \[ f_{t,T} = \mathbb{E}[s_{t+T} | l_t^M] + RP_{t,T} \]
  - Rogoff (2008); Buser, Karolyi & Sanders (OSU, 1996): there seem to be hard to forecast short-term premia

Relationship to Expected Future Spot 4

- Uncovered IRP
  - CIRP
    \[ f_{t,T} = l_t \left( \frac{1 + i \frac{T}{360}}{1 + r \frac{T}{360}} \right) \]
  - + FP
    \[ f_{t,T} = \mathbb{E}[s_{t+T} | l_t^M] \]
  - = UIRP
    \[ \mathbb{E}[s_{t+T}] = l_t \left( \frac{1 + i \frac{T}{360}}{1 + r \frac{T}{360}} \right) \]
Relationship to Expected Future Spot 5

• Uncovered IRP
  • UIRP
    \[ \frac{E[s_{t+1}]}{s_t} - 1 = \frac{l - i}{1 + i} \frac{T}{360} \]
  • Usefulness
    – empirical evidence: depends on horizon
  • Information in the order flow?
    » Evans & Lyons (JPE 2002)
      \[ Citi \text{ order flow data explains 16\% 1-mo ahead (93-99)} \]

Practical B: UIRP and Carry Trades

• Basic idea:
  • What if UIRP does not hold?
    » If IR diff is not reflected in expected currency path
      » Then temptation beckons!
  • Strategies
    » IR diff is big \( \Rightarrow \) borrow low, invest high & bet that the spot
      FX rate doesn’t move much (Case 1)
    » IR diff is very small \( \Rightarrow \) bet on FX rate changes (Case 2)
    » Either way, light up a bunch of candles and pray!
• Examples

UIRP and Carry Trades 2

• **Case 1:** Large IR differentials
  • What if UIRP does not hold?
    » Low exchange rate volatility + persistent interest rate differentials = temptation beckons!
  • Strategy
    » Borrow in the “funding” currency (low-yielding)
    » Invest in the “target” currency (high-yielding)
    » Light up a bunch of candles and pray!
• Examples – *Before Lehman Bros. (BIS, 2008)*
  • Funding: Yen, SF; Target: A$, NZ$, £; also Real, ZAR
UIRP and Carry Trades 4

• March 2009 idea – Russian rouble? (RUB)
  – Situation: rouble was yielding high interest rate
  – Hypothesis: rouble expected stable in next 2-3 months
    – Why? better-than-expected developments in Russia’s current account
      (e.g., imports fell faster than expected)
    – Could it get better? expected rouble to be at least stable in 39-41 official
      range, and potentially 1-2% stronger in a month
  – Strategy:
    • Buy rouble @ 40.3 on a 1-month NDF against dollar-euro basket
    • Look to roll the position once (possible)
  – Risks? Oil prices and global risk appetite, LT prospects
    • “Solution”= set a stop (on spot) at 41.3? (spot is 39.8, range is 39-41)?

UIRP and Carry Trades 5

• What is the catch? Risk!
  • Formal studies
    – Fat tails
  • Pick up pennies in front of a steamroller?

UIRP and Carry Trades 3

• Case 2: Similar IR yet large risk differentials
  • What if UIRP does not hold?
    » Low interest rate differentials + some currencies seem weak = temptation beckons!
  • Strategy
    » Borrow in the weak currency
    » Invest in the strong currency
    » Light up a bunch of candles and pray!
  • Examples – Lehman (Q4-2008 to Q1-2009)
    • Interest rates are similar ⇒ go for “safe” (Yen, USD)
Valuing a Forward Position

- Forward price
  - delivery price
    - price at which the underlying asset will be delivered
    - agreed upon at time forward is entered into
  - forward price
    - delivery price that would make the contract have 0 value
    - changes during life of contract (who should care?)
  - forward price = delivery price
    - when contract is created

Valuing Forward Positions 2

- Valuing an existing forward position
  - enter into position at $K$ (delivery price, i.e., initial fwd price)
  - forward price moves from $K$ to $F$
- value of a long: $f = (F - K) e^{-rT}$
  » went long at 1.52$/€, fwd price up to 1.53, I "made" 1c/€
  » but this difference is only realized at maturity => PV it!
- value of a short: $f = (K - F) e^{-rT}$
  » if the fwd price goes up, the value of a short position falls

International Financial Management

IRP in Real Life
Practice A: Real Life Forward-Spot Parity

• Arbitrage
  • so far: bid and asked rates are the same
  • in reality
    » bid-asked spreads on the markets (or brokerage fees)
    » borrowing costs more than depositing
  • consequence
  • equalities vs. bounds

“Real-life” IRP – Transaction Costs 2

• No-arbitrage condition 1
  \[ F_{t,T}^b = \frac{S_t^b(1+\frac{T}{360})}{(1+\frac{T^b}{360})} \]

• No-arbitrage condition 2
  \[ F_{t,T}^a = \frac{S_t^a(1+\frac{T}{360})}{(1+\frac{T^a}{360})} \]

“Real-life” IRP – Example with Costs 1

Suppose that you are a trader at J.P. Morgan allowed to do arbitrage. Annualized six-month LIBOR for the Yen and the USD are:

Bid (deposit) – Ask (Borrow)
USD Yen
3.50%-3.525%
5.99%-6.025%

You learn that Daiwa will let customers buy and sell Y spot at 100.18.0/0.15. A trader at Barclays is simultaneously quoting bid and ask 6-month swap rates of -300 points (i.e., he will buy and sell 6-month forward at 97.1850/50.1850).

a. Can you make money out of these quotes? Explain thoroughly.
At first sight, it appears that the gain or loss will be very small, since covered IRP roughly holds: the $ is selling at about a 2.5% 6-month forward discount, which is about the 6-month interest rate differential between Japan and the US.

When looking closely at the numbers, though, we see an arbitrage opportunity. To see this, construct the forward rate implied by the interest rate differential and the spot rate:

\[
f = \frac{s}{1+i} = \frac{1+i\cdot180}{1+i\cdot360} = \frac{0.01 \/ 1¥}{1+0.055\cdot180/360} \approx 0.010249 \/ 1¥
\]

Now, compare this with the 6-month forward rate quoted directly by Daiwa: 0.010256 \$/1¥.

Clearly, you will want to buy low and sell high, i.e.:

1. Sell to Daiwa ¥ 6-month outright forward at 97.50¥/1$.
2. Borrow $ at 5.5%.
3. Buy ¥ spot with the borrowed dollars at 100¥/1$.
4. Invest the ¥ at the rate of 0.5%.

\[
\begin{align*}
\text{Cash-flows today} & \quad \text{Cash-flows in 6 months} \\
a. & +1$ & \text{(borrow 1$)} & -0.02795 \text{($loan & interest)} \\
b. & -1$ & \text{(exchange for ¥)} & \text{Ann} \\
c. & +100¥ & \text{(invest ¥)} & +100.25 ¥ \\
d. & -100¥ & \text{(sell ¥ forward)} & +102.25 ¥ \\
e. & 0 & \text{none} & +0.0007 $ \\
\text{Total} & 0 & \text{none} & +0.0007 $ \\
\end{align*}
\]

For every dollar borrowed, the gain is 0.0007 $, i.e., a 0.07% profit margin.
“Real-life” IRP – Example with Costs 5

Question
Suppose that you are a trader of JP Morgan allowed to do arbitrage. From a phone call to a trader at Daiwa Bank, you learn that Daiwa will:

- lend and borrow ¥ at 0.5%-0.625% for 6 months (annualized rates)
- lend and borrow $ at 5.375%-5.5% for 6 months (annualized rates)
- buy and sell ¥ spot at 100.00-50 ¥/$
- buy and sell ¥ 6-month forward at 97.00-50 ¥/$

b. Suppose you must borrow $1m from Daiwa for JP Morgan (e.g., to carry out some unrelated investment strategy). What would your total borrowing cost be?

“Real-life” IRP – Example with Costs 6

Answer
Your borrowing choices are the following:

1. either borrow $ from Daiwa at 5.5%: the total $ cost in 6 months would be $27,000.

2. or create a similar pattern of cash-flows, borrowing in ¥, converting the ¥ into $, and locking in the $ cost of the ¥ loan through a forward contract. Here, the cost would be as follows:
   - you need $1m today, hence you borrow ¥100,500,000 and sell them spot for $1m (i.e., you buy $1m at the asked price of ¥100.50/1$)
   - in 6 months, you will need to pay back ¥100,814,063; you can lock in today the $ cost of this repayment by buying $1,039,320 6-month forward. The total $ cost would be: $39,320. The operation I have just described is called a swap.

Bottom Line: Since borrowing directly in $ is cheaper ($27,000 vs. $39,320), you should borrow $.

Real Life Forward-Spot Parity

• Derivations
  • in the following pages, we derive the no-arbitrage bounds in the presence of bid-asked spreads

• Not exam material
  • those pages are here only for students interested in more theoretical aspects of the relationships used in class; no one will be tested on them
IRP without costs

- $I$
  - $t+T$
    - $S$: $-1$
    - $S_t$: $\frac{1}{1+i_t^T}$
    - $FX$: $\frac{1}{S_t}$
    - $FX$: $\frac{-1}{S_t}$
    - $FX$: $\frac{1}{S_t}$
    - $FX$: $\frac{F_{S,F}}{S_t}$

IRP without costs 2

- Net cost
  - domestic: $\$1$
  - foreign: $\$1$

- Risk
  - similar

- Return equality condition
  $\frac{T}{360} F_{S,F} \frac{1}{S_t} \frac{1}{1+i_t^T}$

IRP without costs 3

- $A$
  - $t+T$
    - $S$: $\$1$
    - $S_t$: $\frac{1}{1+i_t^T}$
    - $FX$: $\frac{1}{S_t}$
    - $FX$: $\frac{-1}{S_t}$
    - $FX$: $\frac{1}{S_t}$
    - $FX$: $\frac{F_{S,F}}{S_t}$
    - $FX$: $\frac{F_{S,F}}{S_t}$
IRP without costs 4

- Net flows
  - t:  FX: 0; $: 0
  - t+T: FX: 0; S: -\frac{T_{FX}}{360} \left( 1 + \frac{T}{360} \right) \frac{S}{S_i}

- No-arbitrage condition
  \frac{T_{FX}}{360} \left( 1 + \frac{T}{360} \right) \frac{S}{S_i} = \frac{T}{360} \left( 1 + \frac{T}{360} \right) 

“Real-life” IRP – Transaction Costs

- 1) borrow S1
- 2a) buy FX spot S: -1
  FX: 1/\frac{S_i^a}{T_{FX}}
- 2b) invest FX
  FX: -\frac{1}{\frac{S_i^b}{T_{FX}}}
- 2c) short FX forward
  FX: \frac{1}{\frac{S_i^c}{T_{FX}}}

“Real-life” IRP – Transaction Costs 2

- Net flows
  - t:  FX: 0; $: 0
  - t+T: FX: 0; S: -\frac{T_{FX}}{360} \left( 1 + \frac{T}{360} \right) \frac{S_i^b}{S_i^a}

- No-arbitrage condition
  \frac{T_{FX}}{360} \left( 1 + \frac{T}{360} \right) \frac{S_i^b}{S_i^a} = \frac{T}{360} \left( 1 + \frac{T}{360} \right) \frac{S_i^a}{S_i^b}
“Real-life” IRP – Transaction Costs 3

- \( t \)
  - \( (1) \) borrow FX 1
  - \( (2a) \) buy $ spot S: \( S^b_t \)
  - \( (2b) \) invest $ S: \(-S^b_t\)
  - \( (2c) \) buy FX forward S: \( S^b_t (1+i_j) \)

\[ F^b_t = \frac{S^b_t}{(1+i_j)_{360}} \]

“Real-life” IRP – Transaction Costs 4

- Net flows
  - \( t \): FX: 0; $: 0
  - \( t+T \): S: 0; FX: \( \frac{S^b_T}{(1+i_j)_{360}} \)

- No-arbitrage condition

\[ (1+i_j)_{360} \frac{S^b_T}{F^b_{T, t}} \geq \frac{S^b_t}{F^b_{t, t}} \]

\[ (1+i_j)_{360} \frac{S^b_T}{F^b_{T, t}} \geq \frac{S^b_t (1+i_j)_{360}}{F^b_{t, t}} \]