A note on Parity Conditions (CIRP, “FP”, UIRP, PPP) and Carry Trades.

Early in the course, we study various parity conditions and the connections between them. This note provides you with a summary of the key relationships.

a. (Covered) Interest Rate Parity.

The first parity condition we will cover is Interest Rate Parity (IRP), which links interest rate differentials between countries to forward discounts or premia (premiums) on the foreign exchange market. Intuitively, IRP says that – assuming there are no exogenous limitations to arbitrage – a country’s currency will sell at a forward premium against the currency of another country when (nominal) interest rates in the first country are lower than they are in the other country. Formally, IRP can be written as:

$$f_{t,T} = s_t \left( 1 + \frac{i_T T}{360} \right) \left( 1 + \frac{i^*_T T}{360} \right) \quad \text{or} \quad \frac{f_{t,T} - s_t}{s_t} = \frac{(i - i^*) \frac{T}{360}}{1 + \frac{i^* T}{360}}$$

for most currencies. For the pound Sterling (£ or GBP) and the Australian dollar (A$ or AUD), the formula is slightly different:

$$f_{t,T} = s_t \left( 1 + \frac{i_T T}{360} \right) \left( 1 + \frac{i^* T}{365} \right) \quad \text{or} \quad \frac{f_{t,T} - s_t}{s_t} = \frac{i_T T - i^* \frac{T}{365}}{1 + \frac{i^* T}{365}}$$

for the £

$$f_{t,T} = s_t \left( 1 + \frac{i_T T}{360} \right) \left( 1 + \frac{i^* A$}{365} \right) \quad \text{or} \quad \frac{f_{t,T} - s_t}{s_t} = \frac{i_T T - i^* \frac{T}{365}}{1 + \frac{i^* A$}{365}}$$

for the A$

Because IRP is derived by arbitrage and involves the use of forward contracts to cover FX risk, it is often called covered interest rate parity (CIRP). Only legal\(^1\) or practical\(^2\) limits on arbitrage may prevent CIRP from holding.

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\(^1\) For example, if a government bans forward transactions or limits their use to specific traders while imposing restrictions on the rates that may be used by those traders.

\(^2\) For example, if it is effectively impossible for a large number of traders to borrow at posted rates, as was the case after the demise of Lehman Brothers in 2008.
b. Forward “Parity” (a.k.a. Speculative Efficiency Hypothesis).

Under the assumption that markets are efficient and that FX market participants are effectively risk-neutral, the forward rate for delivery at a given future date \((f_{t,T})\) should arguably reflect all of today's market information \((I_t^M)\) about the spot rate that investors expect to prevail at that date. That is, the forward rate should be an unbiased predictor of the future spot rate. Formally, this intuition can be written:

\[ f_{t,T} = E[s_{t+T} | I_t^M] = E[s_T] \]

As an empirical mater, however, forward parity does not seem to hold at maturities less than a year – see, e.g., Chinn (2006) for a good and thorough discussion of the usefulness of “forward parity” at different horizons. A key reason is the apparent existence of a time-varying (and hard to predict) risk premium embedded in the forward rate. Consequently, the relationship between forward and expected spot rate should be written more generally as:

\[ f_{t,T} = E[s_{t+T} | I_t^M] + \text{risk premium} = E[s_T] + RP_t \]

In practice, if you were asked to make a prediction for (say) 3 months hence, you might reasonably use this empirical evidence to argue that the forward rate is likely to be a bad forecasting tool of the future spot rate at this (or a shorter) horizon. Indeed, you might reasonably make an argument that the best 3-month forecast is not (unlike what many older international finance textbooks might have suggested) the forward rate but, instead, the current spot rate.

c. (Uncovered) Interest Rate Parity.

Combining covered IRP and forward parity yields the nominal interest rate differentials as a predictor of future exchange rates. The resulting relation is called “Uncovered Interest Rate Parity (UIRP).” The term “uncovered” is used because this relation is not derived by arbitrage and does not involve the use of forward contracts to cover FX risk. Formally, we can write:

\[ E[s_{t+T}] = s_t \left( 1 + \frac{i}{360} \right) \left( 1 + i^* \frac{T}{360} \right) \]

for most currencies, or:
for the pound Sterling and the Australian dollar, respectively. Alternatively, we can use UIRP to predict percentage exchange rate changes:

$$\frac{E_{s_{t+T}} - s_t}{s_t} = \frac{(i - i^*) \frac{T}{360}}{1 + i^* \frac{T}{365}}$$

for most currencies, or, for the pound and the Australian dollar:

$$\frac{E_{s_{t+T}} - s_t}{s_t} = \frac{i \frac{T}{360} - i^* \frac{T}{365}}{1 + i^* \frac{T}{365}} \quad \text{and} \quad \frac{E_{s_{t+T}} - s_t}{s_t} = \frac{i \frac{T}{360} - i^* \frac{T}{365}}{1 + i^* \frac{T}{365}}$$

In words, the expected percentage appreciation of a currency against another is given by the interest rate differential between the two currencies – with the currency that pays a higher interest rate expected to depreciate against the low-interest-rate currency.

Empirically, uncovered IRP works much better at fairly long-term horizons than at horizons of less than a year – again, I suggest that interested students read the papers by Meredith & Chinn (IMFSP, 2004) and by Chinn (JIMF, 2006), copies of which are included with the electronic reference list.

If a market participant does not anticipate UIRP to hold, then s/he may engage in “carry trades”. Pre-Lehman, famous examples included borrowing in Japanese Yen (as a “funding currency” before 2008 and in 2011-2012) to invest in Australian dollars, Brazilian Reals or even South African rands (as “target currencies”).

As the painful experiences of South-East Asian countries (1997-1998), Eastern European countries (2008-2012) and Japan (Fall 2008, Winter 2013) show, however, over the long run such carry trades can be very dangerous – that is, in the long run, UIRP tends to hold (though there have been some exceptions – think of commodity currencies such as the OZ dollar from 2003 until about a year ago).

d. Purchasing Power Parity.

PPP states that the currency of a country that has a higher rate of inflation than another country should depreciate against the currency of the second country. A corollary is that
countries with the same inflation rate should have stable nominal exchange rates over time. Formally, PPP states that:

$$E_{t+T}^{s} = s_t \left(1 + \mu \frac{T}{360}\right) \left(1 + \mu^* \frac{T}{360}\right)$$

or

$$\frac{E_{t+T}^{s} - s_t}{s_t} = \frac{(\mu - \mu^*) \frac{T}{360}}{1 + \mu^* \frac{T}{360}}$$

Most practitioners use the above formula for all currencies. In the case of the £ and the A$, however, some practitioners may use the following formulas instead:

$$E_{t+T}^{s} = s_t \left(1 + \mu \frac{T}{365}\right) \left(1 + \mu^* \frac{T}{365}\right)$$

or

$$\frac{E_{t+T}^{s} - s_t}{s_t} = \frac{\mu \frac{T}{360} - \mu^* \frac{T}{365}}{1 + \mu^* \frac{T}{365}}$$

$$E_{t+T}^{s} = s_t \left(1 + \mu \frac{T}{360}\right) \left(1 + \mu^* \frac{T}{365}\right)$$

or

$$\frac{E_{t+T}^{s} - s_t}{s_t} = \frac{\mu \frac{T}{360} - \mu^* \frac{T}{365}}{1 + \mu^* \frac{T}{365}}$$

e. Fisher effect.

According to the Fisher effect, a country's nominal rate of interest (i) can be decomposed into two components: the real rate of interest (r) and the inflation rate (µ). Formally, we have:

$$(1+i) = (1+r)(1+\mu)$$

and

$$(1+i^*) = (1+r^*)(1+\mu^*)$$

In countries with fairly low inflation rates, the Fisher relation can be simplified to:

$$i = r+\mu$$

and

$$i^* = r^*+\mu^*$$

f. Real Exchange Rate: Linking PPP and UIRP.

The real exchange rate at the period of reference t (s'_t) is by definition equal to the nominal exchange rate at that time (s_t). The real exchange rate T days later (s'_{t+T}) is defined as the nominal exchange rate (s_{t+T}) adjusted for inflation:

$$s'_{t+T} = s_{t+T} \left(1 + \mu^* \frac{T}{360}\right) \left(1 + \mu \frac{T}{360}\right)$$

where, µ and µ* are the annualized inflation rates from t to t+T in the US and the foreign country, respectively. Clearly, if PPP holds, then real FX rates do not change through time.
Using the Fisher effect, we see that if real interest rates are equal across countries, then PPP and UIRP will make similar predictions about the future spot rate.

\[
\frac{E[s_{t+T}] - s_t}{s_t} = \frac{\left\{ (\mu - \mu^*) + (r - r^*) \right\} \frac{T}{360}}{1 + \left( r^* + \mu^* \right) \frac{T}{360}}
\]

Put differently, nominal exchange rate changes will be determined by PPP if current account considerations are the only determinant of the exchange rate; but this will be true only if capital account considerations are not important, i.e., if no "distortions" are introduced by differences in the real interest rates offered by various countries.

References.


Both papers are available from: [http://www.ssc.wisc.edu/~mchinn/research.html](http://www.ssc.wisc.edu/~mchinn/research.html)