What to do with this practice set?

To help students prepare for the assignment and the exams, practice sets with solutions will be handed out. These sets contain worked-out end-of-chapter problems from BKM3 and BKM4. These sets will not be graded, but students are strongly encouraged to try hard to solve them and to use office hours to discuss any problems they may have doing so. One of the best self-tests for a student of his or her command of the material before a case or the exam is whether he or she can handle the questions of the relevant practice sets. The questions on the exam will cover the reading material, and will be very similar to those in the practice sets.

Question 1:

Suppose you invest in zero coupon bonds. One matures in 1 year, paying $100, and its price is $56.93. The other matures in 2 years, paying $1,100, and its price is $943.07.

(a) Compute the yield on each bond.
(b) Compute the duration for each bond.
(c) Compute the weighted-average duration for the portfolio of the two bonds. (NOT Exam Material)
(d) Compute the duration of the portfolio of the two bonds. (NOT Exam Material)

Question 2:

Consider a bond that has a 30-year maturity, an 8% coupon rate, and sells at an initial yield to maturity of 8%. Because the coupon rate equals the yield to maturity, the bond sells at par value: P = $1,000.00. Also, you are told that the modified duration (D*) of the bond, at its initial yield, is 11.26 years, and that the bond’s convexity is 212.4. Suppose that the bond’s yield increases from 8% to 10%.

(a) Predict how much the bond price would decline by applying the duration rule.
(b) You can compute (exactly) that the bond price will actually fall to $811.46, corresponding to a decline of 18.85%. Can you explain differences with the result in item (a)?
(c) Now consider that you are interested in predicting how much the bond price would change by applying the duration-with-convexity rule. How do you analyze the result in this case?
(d) Now consider that there is a much smaller change in bond’s yield of 0.1%, so that the price of the bond would actually fall to $988.85, which corresponds to a decline of 1.115%. Predict how much the bond price would change by applying both the duration and the duration-with convexity rules, and then analyze how the results differ from those in (a) and (c).
Question 3:
Suppose an American call option is written on Nortel stock. The exercise price is $105 and the present value of the exercise price is $100.

(a) What is the hard floor price of the option if Nortel stock sells for $160?
(b) Sketch a graph of the hard floor option prices against the Nortel stock’s price (this is equivalent to plotting the intrinsic value in terms of Nortel’s stock price).
(c) At a stock price of $125, you notice the option selling for $18. Would this option price be an equilibrium price? Explain.
(d) Although the Black-Scholes call option formula was developed for European call options, could it be legitimately used to value these American calls?

Question 4:
A stock is selling today for $150. If you are the recorded owner of the stock today, you will receive a $15 dividend. Someone holding the stock tomorrow (but not today) is not entitled to the dividend. You do hold an American call option on the stock with an exercise price of $130. Suppose the anticipated option price is $12 when the stock price is $135. Would you hold on to your option or exercise it? Why?

Question 5: (NOT Exam Material)
The common stock of the P.U.T.T Corporation has been trading in a narrow price range for the past month, and you are convinced it is going to break far out of that range in the next three months. You do not know whether it will go up or down, however. The current price of the stock is $100 per share, and the price of a 3-month call option at an exercise price of $100 is $10.

(a) If the risk-free interest rate is 10% per year, what must be the price of a 3-month put option on P.U.T.T stock at an exercise price of $100? The stock pays no dividend.
(b) What would be a simple options strategy to exploit your conviction about the stock price’s future movements? How far would it have to move in either direction for you to make a profit on your initial investment?

Question 6:
Consider the following options portfolio: we are on March 23, 2006 and you write an April 2006 maturity call option on IBM with exercise price 85. You write an April 2006 IBM put option with exercise price 75.

(a) Graph the payoff of this portfolio at option expiration as a function of IBM’s stock price at that time.
(b) What will be the profit/loss on this position if IBM is selling at 77 on the option maturity date? What if IBM is selling at 90? Use the Wall Street Journal listing from Figure 20.1 (BKM7 page 694) to answer this question.
(c) At what two stock prices will you just break even on your investment?
(d) What kind of “bet” is this investor making? That is, what must this investor believe about IBM’s stock price in order to justify this position?
Questions from PS#2 that are exam material for the Final

**Question 9:**

Rank the following bonds in order of descending duration:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Coupon (%)</th>
<th>Time to Maturity (Years)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>8</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>E</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

**Question 10:**

Pension funds pay lifetime annuities to recipients. If a firm expects to remain in business indefinitely, its pension obligation will resemble a perpetuity. Suppose, therefore, that you are managing a pension fund with obligations to make perpetual payments of $2 million per year to beneficiaries. The yield to maturity on all bonds is 16%.

(a) If the duration of 5-year maturity bonds with coupon rates of 12% (paid annually) is 4 years and the duration of 20-year maturity bonds with coupon rates of 6% (paid annually) is 11 years, how much of each of these coupon bonds (in market value) will you want to hold to both fully fund and immunize your obligation?

(b) What will be the par value of your holdings in the 20-year coupon bond?
Question 1:

(a) Yield on 1-year bond = ($100/$56.93) – 1 = 75.65%

Yield on a 2-year bond = [($1100/$943.07)^{1/2}] – 1 = 8%

(b) Duration for 1-year bond = 1 year (single payment).
Duration for 2-year bond = 2 years (single payment).

(c) The weighted average duration for the portfolio is equal to:

\[1 \times \frac{56.93}{1000} + 2 \times \frac{943.07}{1000} = 1.943 \text{ years}\]

(d) The duration for the portfolio is equal to:

\[1 \times \frac{100}{1.10}/1000 + 2 \times \frac{1100}{1.10^2}/1000 = 0.90909 + (2 \times 0.90909) = 1.909\]

Question 2:

(a) We know that:

\[(\Delta P/P) = -D\Delta y, \text{ where } D \text{ stands for the modified duration of the bond at its initial yield.}\]

Thus we could predict a price decline of

\[(\Delta P/P) = -11.26 \times 0.02 = -0.2252, \text{ or } -22.52%.\]

(b) –22.52% is considerably a higher price decline than the actual decline of 18.85%. In this case, the duration rule was not a very accurate measure of the sensitivity of bond prices, in the sense that for a 2% yield change, the duration rule underestimated the new value of the bond (i.e., predicted a lower bond price) following such a change in its yield. Thus, duration predicted that the bond price would fall more than it actually fell.

(c) The duration-with-convexity rule is given by

\[(\Delta P/P) = -D\Delta y + (1/2) \times \text{Convexity} \times (\Delta y)^2.\]

Thus,

\[(\Delta P/P) = -11.26 \times 0.02 + (1/2) \times 212.4 \times (0.02)^2 = -0.1827, \text{ or } -18.27% .\]
The predicted decline of -18.27% is far closer to the exact change in the bond price of -18.85%. In this situation, the duration-with-convexity rule is more accurate to predict a higher bond price.

(d) Without accounting for convexity, we would predict a price decline of

\[
\left(\frac{\Delta P}{P}\right) = -11.26 \times 0.001 = -0.01126, \text{ or } -1.126\%.
\]

If we account for convexity, then we will get almost the precisely correct price change of 1.115%:

\[
\left(\frac{\Delta P}{P}\right) = -11.26 \times 0.001 + (1/2) \times 212.4 \times (0.001)^2 = -0.011154, \text{ or } -1.1154\%.
\]

In this case, for a much smaller yield change of 0.1%, convexity would matter less. In other terms, since the change in the bond’s yield is very small, the convexity term, which is multiplied by \((\Delta y)^2\), will be extremely small and will do little to the approximation. Thus, the duration rule is quite accurate in such a situation, even without accounting for convexity. In general, convexity is more important as a practical device when potential interest rate changes are large.

**Question 3:**

(a) Hard floor price = \(V_S - X = 160 - 105 = 55\).

(b) See Fig. 21.1, with \(X=105\$\).

(b) An option price of $18 is below the hard floor price of $20. In this case, everyone would want the call option. You could then acquire a share of Nortel stock for less than the current market price. Simply buy the option (for $18), exercise it (paying $105), and you would then own a share of Nortel for a total price of $123.

(c) The Black-Scholes call option procedure would be legitimate for American options if the stock paid no dividends over the remaining life of the option. This is because the right to exercise prior to maturity has no value.

**Question 4:**

It would make sense to exercise the option now. Exercising the option now, for an exercise price of $130, would give you the $15 dividend, plus a share of stock worth $135, while not exercising would mean you would hold an option worth $12 (tomorrow). Needless to say, having $20 today is a better outcome than having a possible $12 tomorrow.

- Striking today means you exercise your right to buy the stock *cum*-dividend, at the exercise price of $130. After striking, you would *either* resell the stock right away for $150 *or* keep the $15 dividend and still have a share of stock which, tomorrow, should be worth (*ex*-dividend) $135 – either way, your total gain should be $20. Of course, you could simply sell the option for $20 today as well – abstracting from transaction fees, you
should be indifferent between selling the option today or exercising it today – that is, all $20 of the option’s premium today represent intrinsic value.

- By contrast, not exercising would mean you would hold onto the option, which will be worth a mere $12 tomorrow ($5 in intrinsic value and, hence, $7 in time value).

**Question 5:**

(a) From the put-call parity, we know that

\[ P = C - S + \frac{X}{(1 + r)^T}. \]

Thus,

\[ P = $10 - $100 + \left[ \frac{$100}{(1.10)^{1/4}} \right] = $7.645 \]

(b) Purchase both a put and a call on the stock. This strategy is called “buying a straddle.” The total cost of the straddle would be $10 + $7.645 = $17.645, and this is the amount by which the stock would have to move in either direction for the profit on the call or put to cover the investment cost (not including time value of money considerations). If the time value of money were taken into account, the stock price would need to swing in either direction by $17.645 \times (1.10)^{1/4} = $18.07.

**Question 6:**

(a) | S < 75 | 75 < S < 85 | S > 85 |
---|---|---|---|
Written Call | 0 | 0 | - (S - 85) |
Written Put | - (75 - S) | 0 | 0 |
Total | S – 75 | 0 | 85 - S |

(b) Proceeds from writing options (i.e., price obtained by the seller):

- Call = $0.95
- Put = $0.10
- Total = $1.05

Hence, we have:

If IBM sells at $77, both options expire out of the money, and profit equals $1.05 (abstracting from time value of money).

If IBM sells at $90, the call written results in a cash outflow of $5 at maturity, and an overall loss of $1.05 - $5 = $3.95.
(c) You break even when either the put or the call written results in a cash outflow of $1.05. For the put, this would require that \( 1.05 = 75 - S \), or \( S = 73.95 \). For the call, this would require that \( 1.05 = S - 85 \), or \( S = 86.05 \).

(d) The investor is betting that IBM stock price will have relatively low volatility. This position is similar to what is called by traders as a “straddle.”
Solutions to Questions from PS#2 that constitute exam material for the Final

**Question 9:**
C, D, A, B, E.

To see why we can rank-order the bonds without doing computations in the present example, remember that duration is – other things equal otherwise – increasing in time to maturity (TTM) and decreasing in both coupon rate (CR) and yield to maturity (YTM). Here, bond C has the highest TTM and the lowest YTM and CR of all bonds. Hence, it clearly has the highest duration. Bond D is next, as it has the same TTM and YTM but a higher coupon rate than bond C (and a higher TTM, lower YTM, and CR than all the remaining bonds). Proceeding in this way, you get the order C, D, A, B, E.

Note that, in general, bonds cannot be rank-ordered so simply; for example, it is usually hard to avoid using computations to rank two bonds by duration when they have “almost” the same YTM, CR and TTM.

**Question 10:**

(a) • PV of the firm’s “perpetual” obligation = ($2 million/0.16) = $12.5 million.
   • Based on the duration of a perpetuity, the duration of this obligation = (1.16/0.16) = 7.25 years.

Denote by \( w \) the weight on the 5-year maturity bond, which has duration of 4 years. Then,

\[
w x 4 + (1 - w) x 11 = 7.25, \text{ which implies that } w = 0.5357. \text{ Therefore,}
\]

\[
0.5357 x $12.5 = $6.7 \text{ million in the 5-year bond and}
0.4643 x $12.5 = $5.8 \text{ million in the 20-year bond.}
\]

The total invested amounts to $(6.7+5.8) million = $12.5 million, fully matching the funding needs.

(b) The price of the 20-year bond is 60 x PA(16%, 20) + 1000 x PF(16%, 20) = $407.11.

Therefore, the bond sells for 0.4071 times its par value, and

\[
\text{Market value} = \text{Par value} x 0.4071
\]

\[
\Rightarrow \text{ $5.8 \text{ million} = \text{Par value} x 0.4071}
\]

\[
\Rightarrow \text{ Par value} = $14.25 \text{ million.}
\]

Another way to see this is to note that each bond with a par value of $1,000 sells for $407.11. If the total market value is $5.8 million, then you need to buy 14,250 bonds, which results in total par value of $14,250,000.