A Note on Parity Conditions.

Early in the course, we study various parity conditions and the links between them. This note provides you with a summary of the major FX parity relationships. It includes (parts d to f) some parity relationships you may have seen in *International Finance*, using the same notation as for IRP; parts d to f are NOT exam material for *Derivatives*.

**a. (Covered) Interest Rate Parity.**

The first parity condition we will cover is Interest Rate Parity (IRP), which links interest rate differentials between countries to forward discounts or premia (premiums) on the foreign exchange market.

**Intuitively**, IRP says that a country’s currency will sell at a forward premium against the currency of another country when (nominal) interest rates in the first country are lower than they are in the other country.

**Formally**, let \( i \) denote the domestic interest rate, let \( i^* \) denote the foreign interest rate, let \( S_t \) denote the spot FX rate (# of units of local currency per 1 unit of foreign currency), and let \( F_{t,T} \) denote the forward exchange rate (where \( T \) is the delivery date). Then IRP requires that:

\[
F_{t,T} = S_t \left( 1 + \frac{i}{360} \right) \left( 1 + \frac{i^*}{360} \right) \quad \text{or} \quad \frac{F_{t,T} - S_t}{S_t} = \frac{(i - i^*)}{1 + \frac{i^*}{360}}
\]

for most currencies. For the pound Sterling (£ or GBP) and the Australian dollar (A$ or AUD), the formula is slightly different:

\[
F_{t,T} = S_t \left( 1 + \frac{i}{360} \right) \left( 1 + \frac{i^*_{GBP}}{365} \right) \quad \text{or} \quad \frac{F_{t,T} - S_t}{S_t} = \frac{i}{360} - \frac{i^*_{GBP}}{365} \left( 1 + \frac{i^*_{GBP}}{365} \right) \quad \text{for the £}
\]

\[
F_{t,T} = S_t \left( 1 + \frac{i}{360} \right) \left( 1 + \frac{i^*_{AUD}}{365} \right) \quad \text{or} \quad \frac{F_{t,T} - S_t}{S_t} = \frac{i}{360} - \frac{i_{AUD}}{365} \left( 1 + \frac{i_{AUD}}{365} \right) \quad \text{for the A$}
\]

where \( S_t \) denotes the spot FX rate (# of units of other currency per 1 GBP or per 1 AUD), and where \( F_{t,T} \) denotes the forward exchange rate (where \( T \) is the delivery date).

Because IRP is derived by arbitrage and involves the use of forward contracts to cover FX risk, it is often called covered interest rate parity (CIRP).
b. Forward Parity (a.k.a. Speculative Efficiency Hypothesis).

Under the assumption that markets are efficient and that FX market participants are essentially risk-neutral, the forward rate for delivery at a given future date \( f_{t,T} \) should reflect all of today's market information \( I_t^M \) about the spot rate that investors expect to prevail at that date. That is, the forward rate \( f_{t,T} \) should be an unbiased predictor of the future spot rate. Formally, this would be written:

\[
 f_{t,T} = E_t[s_{t+T} | I_t^M] 
\]

As an empirical matter, however, the empirical evidence to date overwhelmingly suggests that forward parity does not hold at maturities less than a year (see, e.g., Chinn (2006) for a thorough discussion of the usefulness of “forward parity” at different horizons). A key reason is the apparent existence of a time-varying (and hard to predict) risk premium embedded in the forward rate. In other words, the relationship between forward and expected spot rate appears to be:

\[
 f_{t,T} = E_t[s_T] + \text{risk premium} 
\]

In practice, therefore, to the extent that you are asked to make a prediction for (say) 3 months hence, the forward is therefore likely to be a bad forecasting tool. Given this empirical reality, some make an argument that the best 3-month forecast is not (unlike what many older international finance textbooks used to suggest) the forward rate but, instead, the current spot forex rate.

c. (Uncovered) Interest Rate Parity.

Combining covered IRP and forward “parity” yields the nominal interest rate differentials as a predictor of future exchange rates. The resulting relation is called “Uncovered Interest Rate Parity (UIRP).” The term “uncovered” is used because this relation is not derived by arbitrage and does not involve the use of forward contracts to cover FX risk. Formally, we can write it as:

\[
 E_t[s_{t+T}] = s_t \left( \frac{1 + i \frac{T}{360}}{1 + i^* \frac{T}{360}} \right) 
\]
for most currencies (See Section a. above for notational conventions), or:

\[ E[s_{t+T}] = s_t \left( 1 + \frac{i}{360} \right) \left( 1 + \frac{i^*}{365} \right) \]

and

\[ E[s_{t+T}] = s_t \left( 1 + \frac{i}{360} \right) \left( 1 + \frac{i^*}{365} \right) \]

for the pound Sterling and the Australian dollar, respectively. Alternatively, we can use UIRP to predict percentage exchange rate changes:

\[ \frac{E[s_{t+T}] - s_t}{s_t} = \frac{(i - i^*)}{1 + \frac{i^*}{360}} \]

for most currencies, or, for the pound and the Australian dollar:

\[ \frac{E[s_{t+T}] - s_t}{s_t} = \frac{i}{360} - \frac{i^*}{365} \]

\[ \frac{E[s_{t+T}] - s_t}{s_t} = \frac{i}{360} - \frac{i^*}{365} \]

Once again, in practice, uncovered IRP works much better at fairly long-term horizons than at horizons of less than a year (see also the papers by Meredith & Chinn (IMFSP, 2006) and by Chinn (JIMF, 2006) on the Online Library). If a market participant does not anticipate UIRP to hold, then s/he may engage in carry trades. As the experiences of South-East Asian countries in 1997-1998 and Eastern European countries in 2008-2009 shows, however, over the long run such carry trades are dangerous – that is, in the long run, UIRP usually holds.

d. Purchasing Power Parity.

PPP states that the currency of a country that has a higher rate of inflation than another country should depreciate against the currency of the second country. A corollary is that countries with the same inflation rate should have stable nominal exchange rates over time. Formally, PPP states that:

\[ E[s_{t+T}] = s_t \left( 1 + \mu \frac{T}{360} \right) \left( 1 + \mu^* \frac{T}{365} \right) \]

or

\[ \frac{E[s_{t+T}] - s_t}{s_t} = \frac{(\mu - \mu^*)}{1 + \mu^* \frac{T}{360}} \]

Most practitioners use the above formula for all currencies. In the case of the £ and the A$, however, some practitioners may use the following formulas instead:
According to the Fisher effect, a country's nominal rate of interest \( i \) can be decomposed into two components: the real rate of interest \( r \) and the inflation rate \( \mu \). Formally, we have:

\[
(1+i) = (1+r)(1+\mu)
\]

In countries with fairly low inflation rates, the Fisher relation can be simplified to:

\[
i = r + \mu \quad \text{and} \quad i^* = r^* + \mu^*
\]

e. Fisher effect.

Using the Fisher effect, we see that if real interest rates are equal across countries, then PPP and UIRP will make similar predictions about the future spot rate.

**f. Real Exchange Rate: Linking PPP and UIRP.**

The real exchange rate at the period of reference \( t \) \( (s_t') \) is by definition equal to the nominal exchange rate at that time \( (s_t) \). The real exchange rate \( T \) days later \( (s_{t+T}') \) is defined as the nominal exchange rate \( (s_{t+T}) \) adjusted for inflation:

\[
s_{t+T}' = s_{t+T} \left( 1 + \frac{\mu^*}{360} \right) \left( 1 + \frac{T}{360} \right)
\]

where, \( \mu \) and \( \mu^* \) are the annualized inflation rates from \( t \) to \( t+T \) in the US and the foreign country, respectively. Clearly, if PPP holds, then real exchange rates do not change through time.

Using the Fisher effect, we see that if real interest rates are equal across countries, then PPP and UIRP will make similar predictions about the future spot rate.
\[
\frac{E[s_{t+T}] - s_t}{s_t} = \frac{\left\{ (\mu - \mu^*) + (r - r^*) \right\} T}{1 + (r^* + \mu^*) \frac{T}{360}}
\]

Put differently, nominal exchange rate changes will be determined by PPP if current account considerations are the only determinant of the exchange rate; but this will be true only if capital account considerations are not important, i.e., if no "distortions" are introduced by differences in the real interest rates offered by various countries.

References.