Derivatives & Risk Management

• Last Week:
  – Introduction
  – Forward fundamentals

• Weeks 1-3: Part I – Forwards
  – Forward fundamentals
  – Fwd price, spot price & expected future spot

Part I: Forwards
Forwards: Fundamentals

• Definition
  • contract calling for delivery of a given asset
  • at a given future date, at a price agreed-upon today
  • no money changes hands today (caveat)

• Market participants & Payoffs (*Who & Why?*)

• Market microstructure (*Where and How?*)
  • OTC market
  • Main underlying assets
  • Forward quotes conventions (*forex market only*)

Forwards 2: Who Trades What?

• Market participants
  • hedgers
    » try to avoid impact of price movements
    » short hedgers: have long underlying position, go short
    » long hedgers: have short underlying position, go long
  • speculators
    » try to profit from price movements
  • traders-arbitrageurs
Forwards 3: Results from Trading

• Payoff at maturity (or expiration or delivery)
  • What matters?
    – long position vs. short position
    – hedged position vs. naked position
      » long hedge vs. short hedge
  • Regrets, anyone?

Forwards 4: Payoffs from Naked Positions

• Definition
  • “Naked”
    – means the forward position holder does not have a position in the underlying asset
    – i.e., neither owns nor owes the underlying asset
  • Interpretation
    – Speculator / Investor
    – US regulatory terminology: “Non-Commercial”
**Profit/Loss from a**

*Long* NAKED Forward Position

How does the Long’s payoff come about?

- **Commodity-settled forward**
  
  Go *long* forward at time \( t \) for delivery at time \( T \)
  
  → at time \( T \), long receives the underlying & pay \( F_{t,T} \)
  
  → at time \( T \), long resells the underlying (spot) for \( S_T \)
  
  → **Long’s profit/loss is** \( S_T - F_{t,T} \)

- **Cash-settled forward**
  
  - *a.k.a “NDF” (Non-Deliverable Forward)*
  
  - No delivery, only cash settlement between Long & Short
    
      » Long gets \( \{ S_T - F_{t,T} \} \) from Short (who gets \( \{ F_{t,T} - S_T \} \))
**Profit/Loss** from a  
*Short*  NAKED Forward Position

How does the Short’s payoff come about?

- **Commodity-settled forward**
  
  Go *short* forward at time $t$ for delivery at time $T$
  
  $\rightarrow$at time $T$, short delivers the underlying & gets $F_{t,T}$
  
  $\rightarrow$short is “naked”, so at time $T$ short must buy the
  underlying (spot) for $S_T$
  
  $\rightarrow$ Short’s profit/loss is $F_{t,T} - S_T$

- **Cash-settled forward**
  
  - No delivery (cash settlement only, between Long & Short)
    
    » Short gets $\{F_{t,T} - S_T\}$ while Long gets $\{S_T - F_{t,T}\}$
Forwards 5: Payoffs from Hedged Positions

• Definition
  • “Hedged”
    – means the forward position holder has a position in the underlying asset
    – i.e., either owns nor owes the underlying
      » Long the underlying → hedge by going short
      » Short the underlying → hedge by going long
  • Interpretation
    – US regulatory terminology: “Commercial”

Payoff from a HEDGED Forward Position

• Hedging with “commodity-settled” contract
  • Examples: Long vs. Short hedge

• Hedging with “cash-settled” contract
  • Equivalence of hedging results
  • Advantages?
  • Examples: Long vs. Short hedge
Payoff from a (commodity-settled) HEDGED Long Position in Underlying

\[ F_{t,T} \]

Cash-Inflow

Payoff independent from \( S_T \), i.e., price of Underlying at Expiration

\[ S_T \]

Payoffs from using NDF to HEDGE a Long Position in Underlying

\[ F_{t,T} \]

Proceeds from spot sale of underlying

Net Cash-Inflow

\[ F_{t,T} \]

Profit from short NDF position

\[ S_T \]
Forwards vs. Options

- A forward contract gives the holder the obligations to buy or sell at a certain price.
- An option contract gives the holder the right (but not the obligation) to buy or sell at a certain price.

Option Payoffs at Maturity Payoffs

- Summary: $X$ = Strike price; $S_T$ = Price of underlying asset at maturity.
Forwards 6: What gets traded?

• Underlying assets
  • Most important ones \textit{(in terms of notional value)}
    – Interest rates
      » Forward Rate Agreements (FRAs)
    – Foreign exchange
      » Outright currency forwards and FX swaps
    – Equities and Commodities
  • Relative importance
    – See semi-annual BIS figures in class handout (dt1920a.pdf)

Forwards 7: How does one trade?

• Forward quotes \textit{(OTC forex market)}
  • bid vs. ask
    » bid = price at which market maker \textit{buys from} customers
    » ask = price at which market maker \textit{sells to} customers
  • outright forward \textit{vs. “FX swap rate” (FX market)}
    » spot \quad 1.2275 - 1.2299 \: $ / 1\£
    » “swap rate” \quad 33 - 46
    » outright forward \quad 1.2308 - 1.2345 \: $ / 1\£
Forwards 8

• Forward quotes (OTC)
  • outright forward vs. “swap rate” (forward premium)
    » spot 1.2275 - 1.2299 $ / 1£
    » “swap rate” 33 - 46
    » outright forward 1.2308 - 1.2345 $ / 1£
  • outright forward vs. “swap rate” (forward discount)
    » spot 1.2275 - 1.2299 $ / 1£
    » “swap rate” 45 - 33
    » outright forward 1.2230 - 1.2266 $ / 1£

Forwards 9

• Swap rate & B-A spread
  • observation
    » subtract swap if discount, add if premium
    » why? (size of B-A spread)
  • explanations
    » risk?
    » liquidity (market depth)?
    » others?
Forwards 10

• Annualizing the forward premium/discount
  • Example: spot $1.2275 / 1£
    3-month outright forward $1.2230 / 1£
  • Swap rate
    » $s = -0.0045 $ / 1£ or discount of 45 “points”
  • Percentage premium/discount
    » $(f-s)/s = -0.0045/1.2275 or -0.37%$
  • Annualized percentage premium/discount
    » $(f-s)/s \times 4 = -0.0147 or -1.47%$

Forwards 11

• Regulation?
• Risks
  • volatility of underlying asset price
  • default
    » why?
• Solution
  • Till 2010: currencies vs. most other assets (futures)
  • Since then: impact of Dodd-Frank Act?
Forwards: Pricing

• Relationship to current spot
  • forward-spot parity for domestic assets
  • forward-spot parity for currencies – (covered) IRP
  • compounding & asset type

• Relationship to expected future spot
  • “forward parity” for domestic assets, incl. commodities
    » expectations theory vs. contango vs. (normal) backwardation
  • “forward parity” for currencies
    » uncovered IRP
  • theory vs. practice

Forward Price

• Formal definitions
  • delivery price
    » price at which the underlying asset will be delivered
    » agreed upon at time forward is entered into
  • forward price
    » delivery price that would make the contract have 0 value
    » changes during life of contract (*but, who cares...*)
  • forward price = delivery price
    » when contract is created
Links between Forward & Spot Prices

• Forward-Spot Parity
  • relationship between fwd price & current spot price
  • basic idea:
    » (1) borrow today
    » (2) buy an asset
    » (3) go short forward on the asset
    » the position should have zero risk & zero cost
    » hence, the rate of return should be zero
  • basic idea vs. complications (dividends, seasonality)

Links between Forward & Spot Prices 2

• Domestic-asset sure-dividend case
  
  • (1) borrow cash  
    • (2) buy asset
      • get cash dividends
    • (3) short asset forward
  • total
  
  • hence: 0 = - S_0(1+r) + S_0 d + F_{0,T}  
    and thus  F_{0,T} = S_0(1 + r - d)
Links between Forward & Spot Prices 3

- Cost-of-Carry relationship
  - Forward-Spot Parity = Futures-Spot Parity

- idea
  - buy forward vs. “buy spot and carry”
  - $F_0$ vs. $S_0(1 + r - d)$

- examples of cost of carry
  - **stock:**
    - $r - d$ (no storage cost, dividend rate $d$ earned)
    - $F_0$ vs. $[S_0 - D'](1 + r)$ (PV of dividend known to be $D'$)
    - $F_0$ vs. $S_0(1 + r) - D$ (future dividend known to be $D$)
  - **commodity:**
    - $r + u$ (storage cost born at rate $u$)
    - $F_0$ vs. $[S_0 + U'](1 + r)$ (PV of storage cost known to be $U'$)

Links between Forward & Spot Prices 4

- Dividends
  - problem
    - varies over time (May vs. other periods)
  - solution
    - theory (no-arbitrage bands) vs. practice (triple witching)

- Generalizations
  - Multi-period: $F_0 = S_0 (1 + r - d)^T$
  - Commodities: $F_0 = S_0 (1 + r + u)^T$ --- really?
    - seasonal variations (e.g., harvests)
    - carrying costs vs. storage costs ($u$) vs. convenience yield
Links between Forward & Spot Prices 5

• Commodities
  • financial assets
    » we must have \( F_0 = S_0(1 + r - d) \)
  • commodities as consumption assets
    – basic idea
    » some commodities are not held as financial assets
    » having the commodity may provide benefits (shortages)
    – situation #1
      » \( F_0 > S_0(1 + r + u) \)
      » arbitrage
    – situation #2
      » \( F_0 < S_0(1 + r + u) \)
      » convenience yield \( y \): defined by \( F_0 = S_0(1 + r + u - y) \)

Links between Forward & Spot Prices 6

• Gold futures example
  • Assume delivery in 1 year
  • Let’s not forget anything:
    » dividends? \( \rightarrow \) lease rate
    » Any convenience yield? unlikely
  • \( F = S (1 + r + u - d) \)
    » suppose \( S = 600/\text{ounce} \), \( r =5\% \), \( d = 1\% \) and \( u=0\% \)
    – \( \rightarrow \) Equilibrium \( F = 600 \times (1 + 0.05 - 0.01) = 624 \)
Links between Forward & Spot Prices 7

- **Domestic-asset sure-dividend case** *(continuous time):*
  - (1) borrow \( S_0 / e^d \) \(- (S_0 / e^d) e^r \)
  - (2) buy asset \(- S_0 / e^d\)
  - & reinvest continuous dividends + asset
  - (3) short asset forward + \( F_0 \)
  - total \(0\)
  - hence: \(0 = -[S_0 e^r / e^d] + F_0\) \textit{and thus} \(F_0 = S_0 e^{(r-d)}\)

Links between Forward & Spot Prices 8

- **Discrete time vs. continuous version**
  - discrete \( F_0 = S_0(1 + r - d) \)
  - continuous \( F_0 = S_0 e^{(r-d)} \)

- **Source of the difference**
  - how much is hedged?
  - how are the dividends reinvested?

- **Does it matter?**
  - Size of difference vs. reasonable character of reinvestment hyp.
  - Domestic assets vs. currencies
Links between Forward & Spot Prices 9

- **Foreign currency case**
  - \( t \)
  - \( T \)
  - (1) borrow local \( S_0 \), \(- S_0 (1+r)\)
  - (2) buy 1 FX unit \(- S_0\), \(+ 1 FX\)
  - (3) short currency forward \(+ F_0 (1+r^*)\), \(- (1+r^*) FX\)
  - total \( 0 \), \(- S_0 (1+r) + F_0 (1+r^*)\)

- **hence:** \( 0 = F_0 (1+r^*) - S_0 (1+r) \) *and thus* \( F_0 = S_0 (1+r) / (1+r^*) \)

Links between Forward & Spot Prices 10

- **Interest rate parity theorem**
  - \( = \) Forward-Spot Parity for currencies
    - basic idea
      \[
      f_{t,T} = s_t \left( \frac{1 + i T}{1 + i^* T} \right) \quad \text{or} \quad \frac{f_{t,T} \cdot s_t}{s_t} = \left( \frac{i - i^*}{1 + i^* T} \right) \frac{T}{360}
      \]
    - key difference with “formula” for domestic assets?
    - examples
Links between Forward & Spot Prices 11

• Question

• The 3-month interest rate in Denmark is currently 3.5%.
• Meanwhile, the equivalent interest rate in England is 6.5%.
• All rates are annualized.
• What should be the annualized 3-month forward discount or premium at which the Danish krone will sell against the pound?

Links between Forward & Spot Prices 12

• Answer

• the Danish krone (DKr)
• should sell at a premium
• against the pound
• approximately equal to
• the interest rate differential between the two countries

\[
\frac{f_{t,T} - s_t}{s_t} = \frac{(i_E - i^{*}_{DKr}) \frac{T}{360}}{1 + i^{*}_{DKr} \frac{T}{360}} = \frac{(6.5\% - 3.5\%) \frac{90}{360}}{1 + 3.5\% \frac{90}{360}} = 0.7435\%
\]
Links between Forward & Spot Prices

- Forward & expected future spot
  - At maturity: Convergence property
  - Prior to maturity: Risk premium
    - Equities
    - Commodities:
      » contango vs. backwardation
      » academics vs. practitioners
    - Currencies:
      » One hypothesis: forward parity (no risk premium)
      » In reality: ST(carry trades) vs. LT (UIRP)

Convergence of Forward to Spot

As a forward contract approaches maturity (expiration), the forward price should converge to the spot price
Links between Forward & Spot Prices 14

- Forward & expected future spot (general case)
  - speculators choose between
    » PV of the forward price, known for sure: \( F_0 e^{-r} \)
    » PV of the future spot price, risky: \( E[S_T] e^{-k} \)

- General case: \( F_0 = E[S_T] e^{(r-k)} < E[S_T] \)
  » Why? \( k > r \) if underlying has positive systematic risk

- Special case: \( F_0 = E[S_T] \) (aka expectation hypothesis)
  » if speculators are risk neutral
  » or if the underlying asset is uncorrelated with the market

Links between Forward & Spot Prices 15

- Comparison
  - Forward & current spot
    - discrete time: \( F_0 = S_0 (1 + r - d) \) or \( S_0 (1 + r + u - y) \)
    - continuous time: \( F_0 = S_0 e^{(r-d)} \) or \( S_0 e^{(r+u-y)} \)
  - Forward & expected future spot
    - discrete time: \( F_0 \approx E[S_T] (1 + r - k) \)
    - continuous time: \( F_0 \approx E[S_T] e^{(r-k)} \)
Contango vs. Backwardation

• Forward & current spot (practitioners)
  – Contango: \( S_0 < F_{0,1} < F_{0,2} < \ldots < F_{0,T} < \ldots \)
    • “backdated” contracts are more expensive than “near-month(s)”
    • typical for commodities with positive cost-of-carry
  – Backwardation: \( S_0 > F_{0,1} > F_{0,2} > \ldots > F_{0,T} > \ldots \)

• Forward & expected future spot (Keynes & academics)
  – Contango: \( F_{0,T} > E[S_T] \)
  – (“normal”) Backwardation: \( F_{0,T} < E[S_T] \)
    • “natural” for producers to hedge → pay an “insurance” premium

Contango vs. Backwardation 2

• Contango: \( S_0 < F_{0,T} < F_{0,T+1} < F_{0,T+2} < \ldots \)
  • typical for commodities when cost of carry > 0
    » \( F_{0,T} = S_0 (1 + r - d)^T < F_{0,T+1} = S_0 (1 + r - d)^{T+1} \)
  • if the “term-structure” slope is steep, then storing the commodity becomes profitable
    – Limits? Storage capacity (example: North Sea tankers)
    – Risk? Inelastic supply & demand in the ST
      » convenience yield may rise
Contango vs. Backwardation 3

• Crude, 2004-5:

Parity Conditions for Currencies

• “Forward Parity”
  • speculative efficiency
    » hypothesis vs. arbitrage
    \[ f_{t,T} = E[s_{t+T} | I_t^M] \]
    » is efficiency consistent with a risk premium?
    \[ f_{t,T} = E[s_{t+T} | I_t^M] + RP_{t,T} \]
    » Buser, Karolyi & Sanders (JFI 96): short-term premia
Parity Conditions for Currencies 2

• Uncovered IRP
  • CIRP
    \[ f_{t,T} = s_t \left( \frac{1 + i^* \frac{T}{360}}{1 + i \frac{T}{360}} \right) \]
  • + FP
    \[ f_{t,T} = H[s_{t+T} | t^M] \]
  • = UIRP
    \[ E[s_{t+T}] = s_t \left( \frac{1 + i^* \frac{T}{360}}{1 + i \frac{T}{360}} \right) \]

Parity Conditions for Currencies 3

• Uncovered IRP
  • UIRP
    \[ \frac{E[s_{t+T}] - s_t}{s_t} = \frac{(i - i^*) \frac{T}{360}}{1 + i^* \frac{T}{360}} \]
  • Usefulness
    – empirical evidence: depends on horizon
    » Meredith and Chinn (NBER; IMF SP 2004)
UIRP vs. Carry Trades

• Basic idea:
  • What if UIRP does not hold?
    » If IR diff is not reflected in expected currency path
    » Then temptation beckons for traders!

• Strategies
  » IR diff is big → borrow low, invest high & bet that the spot
    FX rate doesn’t move much (Case 1)
  » IR diff is very small → bet on FX rate changes (Case 2)
  » Either way, light up a bunch of candles and pray!

• Examples

UIRP and Carry Trades 2

• **Case 1:** Large IR differentials
  • What if UIRP does not hold?
    » Low exchange rate volatility + persistent interest rate
differentials = temptation beckons!
  • Strategy
    » Borrow in the “funding” currency (low-yielding)
    » Invest in the “target” currency (high-yielding)
    » Light up a bunch of candles and pray!

• Examples – **Before Lehman crisis**
  • Yen (funding) → A$, NZ$, £, Real, ZAR (targets)
  • Swiss Franc (funding) → Florint, Euro (St Tropez!), Kuna
UIRP and Carry Trades 5

- What is the catch? *Risk!*
  - Formal studies
    - Fat tails
  - Pick up pennies in front of a steamroller?

UIRP and Carry Trades 3

- **Case 2:** Similar IR yet large risk differentials
  - What if UIRP does not hold?
    - Low interest rate differentials + some currencies seem weak = temptation beckons!
  - Strategy
    - Borrow in the weak currency
    - Invest in the strong currency
    - Light up a bunch of candles and pray!
  - Examples – *during Lehman crisis (Q4’08 to Q1 ’09)*
    - Interest rates are similar, so go for “safe”! (Yen, USD)
Valuing Forward Positions

• Forward price
  – delivery price
    – price at which the underlying asset will be delivered
    – agreed upon at time forward is entered into
  – forward price
    – delivery price that would make the contract have 0 value
    – changes during life of contract (who should care?)
  – forward price = delivery price
    – when contract is created

Valuing Forward Positions 2

• Valuing an existing forward position
  – enter into position at $K$ (delivery price, i.e., initial fwd price)
  – forward price moves from $K$ to $F$

• value of a long: $f = (F - K) e^{-rT}$
  » long at 5c/bushel, fwd price goes up to 6cpb, I “made” 1cpb
  » but this difference is only realized at maturity $\rightarrow$ PV it!

• value of a short: $f = (K - F) e^{-rT}$
  » if the fwd price goes up, the value of a short position falls
Part I extra: Forwards

Real Life Forward-Spot Parity

• Arbitrage
  • so far: bid and asked rates are the same
  • in reality
    » bid-asked spreads on the markets (or brokerage fees)
    » borrowing costs more than depositing

• consequence
  • equalities vs. bounds
“Real-life” IRP – Transaction Costs

• No-arbitrage condition 1

\[ F_{t,T}^b \leq S_t^a \frac{(1+i_a T_a)}{(1+i_b \frac{T_b}{360})} \]

• No-arbitrage condition 2

\[ F_{t,T}^a \geq S_t^b \frac{(1+i_b T_b)}{(1+i_a \frac{T_a}{360})} \]

“Real-life” IRP – Example with Costs

Suppose that you are a trader of JP Morgan allowed to do arbitrage. Annualized six-month LIBOR for the Yen and the U.S. dollar are:

- Bid (deposit) – Ask (Borrow) ¥ at 0.5%-0.625%
- Bid (deposit) – Ask (Borrow) $ at 5.375%-5.5%

From a phone call to a trader at Daiwa Bank, you learn that Daiwa will let customers buy and sell ¥ spot at 100.00-50 ¥/$. A trader at Barclays is simultaneously quoting bid and ask 6-month swap rates of –300 points (i.e., he will buy and sell ¥ 6-month forward at 97.00-50 ¥/$).

a. Can you make money out of these quotes? Explain thoroughly.
“Real-life” IRP – Example with Costs 2

Answer

At first sight, it appears that the gain or loss will be very small, since covered IRP roughly holds: the $ is selling at about a 2.5% 6-month forward discount, which is about the 6-month interest rate differential between Japan and the US.

When looking closely at the numbers, though, we see an arbitrage opportunity. To see this, construct the forward rate implied by the interest rate differential and the spot rate:

\[ f = \frac{1+i}{1+\frac{i}{360}} \cdot \frac{180}{360} = 0.010249 \frac{\$}{¥} \]

Now compare this with the 6-month forward rate quoted directly by Daiwa: 0.010256 $ / ¥.

“Real-life” IRP – Example with Costs 3

Clearly, you will want to buy low and sell high, i.e.:

sell to Daiwa ¥ 6-month outright forward at 97.50¥/$.

The other sides of the transaction are:

borrow $ at 5.5%,

buy ¥ spot with the borrowed dollars at 100¥/$,

and invest the ¥ at the rate of 0.5%.
**“Real-life” IRP – Example with Costs 4**

<table>
<thead>
<tr>
<th>cash-flows today</th>
<th>cash-flows in 6 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. + 1$ (borrow 1$)</td>
<td>- 1.0275$ (loan &amp; interest)</td>
</tr>
<tr>
<td>b. - 1$ (exchange for ¥)</td>
<td>none</td>
</tr>
<tr>
<td>e. none (sell ¥ forward)</td>
<td>none</td>
</tr>
<tr>
<td>d. - 100¥ (invest ¥)</td>
<td>+ 100.25 ¥</td>
</tr>
<tr>
<td></td>
<td>- 100.25 ¥</td>
</tr>
<tr>
<td></td>
<td>+ 1.0282 $</td>
</tr>
<tr>
<td>total</td>
<td>+ 0.0007 $</td>
</tr>
</tbody>
</table>

For every dollar borrowed, the gains are 0.0007$, i.e., a 0.07% profit margin.

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**“Real-life” IRP – Example with Costs 5**

**Question**

Suppose that you are a trader of JP Morgan allowed to do arbitrage. From a phone call to a trader at Daiwa Bank, you learn that Daiwa will:

- lend and borrow ¥ at 0.5%-0.625% for 6 months (annualized rates)
- lend and borrow $ at 5.375%-5.5% for 6 months (annualized rates)
- buy and sell ¥ spot at 100.00-50 ¥/1$
- buy and sell ¥ 6-month forward at 97.00-50 ¥/1$

b. Suppose you **must** borrow $1m from Daiwa for JP Morgan (e.g., to carry out some unrelated investment strategy). What would your total borrowing cost be?
“Real-life” IRP – Example with Costs 6

Answer

Your borrowing choices are the following:

(1) either **borrow $ from Daiwa** at 5.5%: the total $ cost in 6 months would be $27,500.

(2) or create a similar pattern of cash-flows, borrowing in ¥, converting the ¥ into $, and locking in the $ cost of the ¥ loan through a forward contract. Here, the cost would be as follows:

- you need $1m today, hence you borrow ¥100,500,000 and sell them spot for $1m (i.e., you buy $1m at the asked price of ¥100.50/1$)

- in 6 months, you will need to pay back ¥100,814,063; you can lock in today the $ cost of this repayment by buying $1,039,320 6-month forward. The total $ cost would be: $39,320. The operation I have just described is called a swap.

**Bottom Line:** Since borrowing directly in $ is cheaper ($27,000 vs. $39,320), you should borrow $.

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Real Life Forward-Spot Parity

- **Derivations**
  - in the following pages, we derive the no-arbitrage bounds in the presence of bid-asked spreads

- **Not exam material**
  - those pages are here only for students interested in more theoretical aspects of the relationships used in class; no one will be tested on them
IRP without costs

- \( t \)
  - invest domestically: $(-1)
  - invest abroad: $(-1)
  - FX: \( S_t \)
  - FX: \( -1/S_t \)

- \( t+T \)
  - $:(1+i\frac{T}{360})$
  - \( F_{t,T} \):
    - $:(1+i\frac{T}{360})$

IRP without costs 2

- Net cost
  - domestic: $1$
  - foreign: $1$

- Risk
  - similar

- Return equality condition
  \[
  (1+i\frac{T}{360}) = \frac{F_{t,T}}{S_t} \cdot (1+i\frac{T}{360})
  \]
IRP without costs 3

- \( t \)
  - (1) borrow $1
  - (2a) buy spot $: -1
  - (2b) invest FX FX: - 1/S_t
  - (2c) short FX forward

\[
\begin{align*}
(1+i^* \frac{T}{360}) & = \frac{F_{t,T}}{S_t}(1+i^* \frac{T}{360}) \\
(1+i^* \frac{T}{360}) & = \frac{F_{t,T}^{-S_t} - S_t}{S_t(1+i^* \frac{T}{360})}
\end{align*}
\]

- \( t+T \)
  - FX: 1/S_t
  - $: (1+i^* \frac{T}{360})

IRP without costs 4

- Net flows
  - \( t \): FX: 0; $: 0
  - \( t+T \): FX: 0; $: -(1+i^* \frac{T}{360}) + \frac{F_{t,T}}{S_t}(1+i^* \frac{T}{360})

- No-arbitrage condition
  \[
  \frac{\frac{F_{t,T}}{S_t} - S_t}{S_t} = \frac{(i-i^*) \frac{T}{360}}{(1+i^* \frac{T}{360})}
  \]
“Real-life” IRP – Transaction Costs

<table>
<thead>
<tr>
<th></th>
<th>( t )</th>
<th>( t+T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>• (1) borrow</td>
<td>$1</td>
<td>$\left(1+i_a \frac{T}{360}\right))</td>
</tr>
<tr>
<td>• (2a) buy FX spot</td>
<td>$: -1 \quad \text{FX:} \frac{1}{S_t^a} )</td>
<td>$: \left(1+i_b \frac{T}{360}\right))</td>
</tr>
<tr>
<td>• (2b) invest FX</td>
<td>( \text{FX:} -1/S_t^a )</td>
<td>( \text{FX:} \frac{1}{S_t^a} \left(1+i_b \frac{T}{360}\right))</td>
</tr>
<tr>
<td>• (2c) short FX forward</td>
<td>( \text{FX:} \frac{1}{S_t^a} \left(1+i_b \frac{T}{360}\right))</td>
<td>( S_t^a \left(1+i_b \frac{T}{360}\right))</td>
</tr>
</tbody>
</table>

“Real-life” IRP – Transaction Costs 2

• Net flows

  • \( t \): \( \text{FX:} 0; \ $: 0 \)
  • \( t+T \): \( \text{FX:} 0; \ $: \left(1+i_a \frac{T}{360}\right) + \frac{F_{t,T}^b}{S_t^a} \left(1+i_b \frac{T}{360}\right)\)

• No-arbitrage condition

\[
(1+i_a \frac{T}{360}) \geq \frac{F_{t,T}^b}{S_t^a} \left(1+i_b \frac{T}{360}\right) \quad \quad \quad \frac{S_t^a \left(1+i_a \frac{T}{360}\right)}{F_{t,T}^b} \leq \frac{\left(1+i_b \frac{T}{360}\right)}{(1+i_b \frac{T}{360})}
\]
“Real-life” IRP – Transaction Costs 3

- \( t \)
  - (1) borrow
    - FX: 1
  - (2a) buy $ spot
    - $: \( S^b_t \)
    - FX: -1
  - (2b) invest $
    - $: \(-S^b_t\)
  - (2c) buy FX forward
    - $: \(-S^b_t(1+i_bT_{360})\)
    - FX: \( \frac{S^b_t}{F^a_{t,T}}(1+i_aT_{360}) \)

- \( t+T \)
  - FX: \(-\frac{T_{360}}{1+i_a} \)

“Real-life” IRP – Transaction Costs 4

- Net flows
  - \( t \):
    - FX: 0; $: 0
  - \( t+T \):
    - $: 0; FX: \(-\frac{S^b_t}{F^a_{t,T}}(1+i_bT_{360})+\frac{S^b_t}{F^a_{t,T}}(1+i_aT_{360}) \)

- No-arbitrage condition

\[
(1+i_aT_{360}) \geq \frac{S^b_t}{F^a_{t,T}}(1+i_bT_{360}) \]
\[
\frac{S^b_t}{F^a_{t,T}}(1+i_aT_{360}) \geq (1+i_bT_{360}) \]