Derivatives & Risk Management

- Previous lecture set:
  - Futures vs. forwards
  - Stock Index Futures

- This lecture set – Part III
  - Interest-Rate Derivatives
    * FRAs
    * T-bills futures & Euro$ Futures

Part III:
Interest Rate Derivatives
Derivatives “of Interest”

- **Interest-Rate Derivatives**
  - Contracts on short-term interest rates
    - FRAs, Eurodollar futures *(also, T-bills futures)*
    - (Single-currency) Interest-rate (IR) Swaps
  - Futures on long-term interest rates
    - e.g., T-bonds & T-notes futures, Bund futures

- **Currency derivatives**
  - Forwards and futures on FX; FX swaps
  - Currency swaps (= *cross-currency interest-rate swaps*)

- **Relative importance:** ISDA + BIS figures

Forward Interest Rates & FRA’s

- **Background**
  - bond pricing
  - term structure of interest rates & pure yield curve
  - forward interest rate *(aka implied forward short rate)*

- **Forward rate agreements**
  - market microstructure
  - locking in rates with FRA’s
Bond Pricing

- Equation for a **coupon** bond:
  
  \[ P = PV(\text{annuity}) + PV(\text{final payment}) \]

  \[ = \sum_{t=1}^{T} \frac{\text{coupon}}{(1+ytm)^t} + \frac{\text{Par}}{(1+ytm)^T} \]

- Terminology: \( T \) = maturity; \( ytm \) = yield to maturity

- Example: \( C_t = $40 \); \( \text{Par} = $1,000 \); disc. rate = 4%; \( T = 60 \)

  \[ P = \sum_{t=1}^{60} \frac{$40}{(1+0.04)^t} + \frac{$1,000}{(1+0.04)^{60}} = $904.94 + $95.06 = $1,000 \]

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Bond Pricing 2

- Equation for a **zero**-coupon bond:
  
  \[ P = PV(\text{final payment}) \]

  \[ = \frac{\text{Par}}{(1+y)^T} \]

- Terminology: \( y \) = T-year spot rate

- Example: \( C_t = $0 \); \( \text{Par} = $1,000 \); disc. rate = 4%; \( T = 60 \)

  \[ P = \frac{$1,000}{(1+0.04)^{60}} = $95.06 \]
Bond Pricing 3

• Why focus on zeroes?
  • The \( ytm \) of coupon bonds is an average of the spot rates of each of the cash flows (idea: reinvestment)

\[
P = \sum_{i=1}^{T} \frac{\text{Coupon}_t}{(1+y_{tm})^T} \frac{\text{Par}}{(1+y_{tm})^T} = \sum_{i=1}^{T} \frac{\text{Coupon}_t}{(1+y_t)^T} \frac{\text{Par}}{(1+y_T)^T}
\]

• The \( ytm \) of zeroes (i.e., the spot) is not corrupted by these reinvestment issues

Term Structure of Interest Rates

• Basic question
  • link between spot rates (= \( ytm \) on zeroes) & maturity

• Bootstrapping short rates from strips
  • forward rates and expected future short rates

• Interpreting the term structure
  • does the term structure contain information?
    • certainty vs. uncertainty

• Recovering short rates from coupon bonds
“Term”inology

• Term structure = yield curve
  • = plot of the YTM as a function of bond maturity
    – Pure yield curve (*special case*)
      • = plot of the spot rate by time-to-maturity

• Short rate vs. spot rate
  • both are “zero rates”
  • 1-period rate vs. multi-period yield (*BKM4 Fig. 14.3*)
  • spot rate = current rate appropriate to discount a cash-flow of a given maturity

Extracting Info re: Short Interest Rates

• From zeroes
  • non-linear regression analysis
  • bootstrapping

• From coupon bonds (*NOT Exam Material*)
  • system of equations
  • regression analysis (no measurement errors)

• Certainty vs. uncertainty
  • forward rate vs. expected future (spot) short rate
(Implied) Forward Interest Rates

• Definition #1
  • forward interest rate for a given period in the future
  • = interest rate implied by current spot rates

• Definition #2
  • “break-even rate”
  • that equates the payoffs of roll-over and LT strategies

Bootstrapping Fwd Rates from Zeroes

• Forward rate
  • “break-even rate”
  • equating the payoffs of ST roll-over vs. LT strategies
  • \( n \) years @ \( y_n \) vs. \((n-1)\) years @ \( y_{n-1} \) plus one year at \( f_n \)
    \[
    n^*y_n = (n-1)^*y_{n-1} + 1^*f_n
    \]

• Intuitive formula
  • \( f_1 = y_1 \) and \( f_n = n^*y_n - (n-1)^*y_{n-1} \)
Bootstrapping Fwd Rates from Zeroes 2

• Forward rate
  • “break-even rate”
  • equating the payoffs of ST roll-over vs. LT strategies
  • \( n \) years @ \( y_n \) vs. \( (n-1) \) years @ \( y_{n-1} \) plus one year at \( f_n \)
    \[
    (1 + y_n)^n = (1 + y_{n-1})^{n-1}(1 + f_n)
    \]

• Precise formula
  • \( f_1 = y_1 \) and \( f_n = \frac{(1 + YTM_n)^n}{(1 + YTM_{n-1})^{n-1}} - 1 \)

Bootstrapping Fwd Rates from Zeroes 3

• Example 1:
  • BKM4 Table 14.2 & Fig.14.1; BKM9 T15.1 & Fig.15.3
  • 4 bonds, all zeroes (reimbursable at par of $1,000)

<table>
<thead>
<tr>
<th>T (maturity)</th>
<th>Price</th>
<th>YTM (spot rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$925.93</td>
<td>8%</td>
</tr>
<tr>
<td>2</td>
<td>$841.75</td>
<td>8.995%</td>
</tr>
<tr>
<td>3</td>
<td>$758.33</td>
<td>9.66%</td>
</tr>
<tr>
<td>4</td>
<td>$683.18</td>
<td>9.993%</td>
</tr>
</tbody>
</table>
Bootstrapping Fwd Rates from Zeroes

- Forward interest rate for year 1
  
  \[ \frac{\$925.93}{(1+f_1)} = \frac{\$1,000}{(1+y_1)} \Rightarrow f_1 = y_1 = 8\% \]

- Forward interest rate for year 2
  
  \[ \frac{\$841.75}{(1+f_1)(1+f_2)} = \frac{\$1,000}{(1+8\%)(1+f_2)} = \frac{\$925.93}{(1+f_2)} \]
  \[ \Rightarrow f_2 = 10\% \]

Bootstrapping Fwd Rates from Zeroes

- Forward rates for years 3 and 4
  
  - keep applying the method
  - you find \( f_3 = 11\% = f_4 \)

- General Formula
  
  \[ f_1 = YTM_1 \]

  \[ 1 + f_n = \frac{(1 + YTM_n)^n}{(1 + YTM_{n-1})^{n-1}} \]
### Bootstrapping Fwd Rates from Zeroes

#### Example 2: Intuitive ("quick & dirty") forward rates

<table>
<thead>
<tr>
<th>Zero-Coupon Rates</th>
<th>Bond Maturity</th>
<th>(1yr) Fwd Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1 = 12.00%$</td>
<td>1</td>
<td>$f_1 = y_1 = 12%$</td>
</tr>
<tr>
<td>$y_2 = 11.75%$</td>
<td>2</td>
<td>$f_2 \approx 11.5%$</td>
</tr>
<tr>
<td>$y_3 = 11.25%$</td>
<td>3</td>
<td>$f_3 \approx 10.25%^*$</td>
</tr>
<tr>
<td>$y_4 = 10.00%$</td>
<td>4</td>
<td>$f_4 \approx 6.25%^*$</td>
</tr>
<tr>
<td>$y_5 = 9.25%$</td>
<td>5</td>
<td>$f_5 \approx 6.25%^*$</td>
</tr>
</tbody>
</table>

*: If computed exactly, $f_3 = 10.26\%; f_4 = 6.33\%; f_5 = 6.30\%$ (we'll show this below)

#### Example 2: "Formal" forward rates

<table>
<thead>
<tr>
<th>Zero-Coupon Rates</th>
<th>Bond Maturity</th>
<th>(1yr) Fwd Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1 = 12.00%$</td>
<td>1</td>
<td>$f_1 = y_1 = 12%$</td>
</tr>
<tr>
<td>$y_2 = 11.75%$</td>
<td>2</td>
<td>$f_2 = 11.5%$</td>
</tr>
<tr>
<td>$y_3 = 11.25%$</td>
<td>3</td>
<td>$f_3 = 10.26%^*$</td>
</tr>
<tr>
<td>$y_4 = 10.00%$</td>
<td>4</td>
<td>$f_4 = 6.33%^*$</td>
</tr>
<tr>
<td>$y_5 = 9.25%$</td>
<td>5</td>
<td>$f_5 = 6.30%^*$</td>
</tr>
</tbody>
</table>

*: If computed quickly, $f_3 = 10.25\%; f_4 = 6.25\%; f_5 = 6.25\%$
Fwd Rate & Expected Future Short Rate

• **Q:** Does IFR equal expected short? (is $f_t = r_t$?)
• **A:** Interpreting the yield curve under uncertainty
  – Short perspective *(often observed $\rightarrow$ exam material)*
    – liquidity preference theory (investors)
    – liquidity premium theory (issuer)
  – *Others: NOT Exam Material*
    • Expectations hypothesis
    • Long perspective
    • Market Segmentation *vs.* Preferred Habitat

Fwd Rate & Exp. Future Short Rate 2

• Short perspective
  • liquidity preference theory (“short” investors)
    » investors need to be induced to buy LT securities
    » example: 1-year zero at 8% vs. 2-year zero at 8.995%
  • liquidity premium theory (issuer)
    » issuers prefer to lock in interest rates
  • $f_2 \geq E[r_2]$
  • $f_2 = E[r_2] +$ liquidity *(or risk)* premium
Fwd Rate & Exp. Future Short Rate 3

• Long perspective \textit{(NOT Exam Material)}
  • “long investors” wish to lock in rates
    » roll over a 1-year zero at 8%
    » or lock in via a 2-year zero at 8.995%
  • $E[r_2] = f_2$
  • $f_2 = E[r_2]$ - liquidity \textit{(or risk)} “premium”

Fwd Rate & Exp. Future Short Rate 4

• Expectation Hypothesis \textit{(NOT Exam Material)}
  • risk premium = 0 and $E[r_2] = f_2$
  • idea: “arbitrage”

• Market segmentation theory \textit{(NOT Exam Material)}
  • idea: clienteles
    » ST and LT bonds are not substitutes
  • reasonable?

• Preferred Habitat Theory \textit{(NOT Exam Material)}
  • investors do prefer some maturities
  • temptations exist
Fwd Rate & Exp. Future Short Rate 5

• In practice
  • liquidity preference + preferred habitat
    » hypotheses have the edge

• Example 2 (continued)

Fwd Rate & Exp. Future Short Rate 6

• Example 2: “Quick & dirty” forward rates

<table>
<thead>
<tr>
<th>Zero-Coupon Rates</th>
<th>Bond Maturity</th>
<th>(1yr) Fwd Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>y₁ = 12.00%</td>
<td>1</td>
<td>f₁ = y₁ = 12%</td>
</tr>
<tr>
<td>y₂ = 11.75%</td>
<td>2</td>
<td>f₂ ≈ 11.5%</td>
</tr>
<tr>
<td>y₃ = 11.25%</td>
<td>3</td>
<td>f₃ ≈ 10.25%*</td>
</tr>
<tr>
<td>y₄ = 10.00%</td>
<td>4</td>
<td>f₄ ≈ 6.25%*</td>
</tr>
<tr>
<td>y₅ = 9.25%</td>
<td>5</td>
<td>f₅ ≈ 6.25%*</td>
</tr>
</tbody>
</table>

*: If computed exactly, f₃ = 10.26%; f₄ = 6.33%; f₅ = 6.30% (we’ll show this below)
Example 2: “Quick” expected future short rates

<table>
<thead>
<tr>
<th>Period</th>
<th>(1yr) Fwd Rate</th>
<th>Expected short rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( f_1 = y_1 = 12% )</td>
<td>( N-A )</td>
</tr>
<tr>
<td>2</td>
<td>( f_2 \approx 11.5% )</td>
<td>( E(y_1') = r_2 \approx 11% )</td>
</tr>
<tr>
<td>3</td>
<td>( f_3 \approx 10.25% )</td>
<td>( E(y_1''') = r_3 \approx 9.75% )</td>
</tr>
<tr>
<td>4</td>
<td>( f_4 \approx 6.25% )</td>
<td>( E(y_1''') = r_4 \approx 5.75% )</td>
</tr>
<tr>
<td>5</td>
<td>( f_5 \approx 6.25% )</td>
<td>( E(y_1''''') = r_5 \approx 5.75% )</td>
</tr>
</tbody>
</table>

*: Assumes a constant 0.5% per year liquidity premium

Example 3:
- short term rates: \( r_1 = r_2 = r_3 = 10\% \)
- liquidity premium = constant 1% per year

\( y_1 = r_1 = 10\% \)

\[
y_2 = \sqrt{(1 + r_1)(1 + f_2)} - 1 = \sqrt{(1 + 10\%)(1 + 10\% + 1\%)} - 1 = 10.5\%
\]

\( y_3 = \frac{1}{3} \sqrt{(1 + r_1)(1 + f_2)(1 + f_3)} - 1 = \frac{1}{3} \sqrt{(1 + 10\%)(1 + 11\%)(1 + 11\%)} - 1 = 10.67\% \)
Measurement: Zeroes vs. Coupon Bonds

- **Zeroes**
  - ideal
  - lack of data may exist (need zeroes for all maturities)

- **Coupon Bonds** *(Next 4 pages NOT Exam Material)*
  - plentiful
  - coupons and their reinvestment
    - low coupon rate vs. high coupon rate
    - short term rates → they may have different YTM

Measurements with Coupon Bonds

- **Example**
  - short rates are 8% & 11% for years 1 & 2; certainty
  - 2-year bonds; Par = $1,000; coupon = 3% or 12%

  - **Bond 1:**
    \[
    \frac{30}{(1+8\%)} + \frac{1,030}{(1+8\%)(1+11\%)} = 894.78 \Rightarrow YTM = 8.98\%
    \]

  - **Bond 2:**
    \[
    \frac{120}{(1+8\%)} + \frac{1,120}{(1+8\%)(1+11\%)} = 1,053.87 \Rightarrow YTM = 8.94\%
    \]
Measurements with Coupon Bonds 2

• Example
  • 2-year bonds; Par = $1,000; coupon = 3% or 12%
  • Prices: $894.78 (coupon = 3%); $1,053.87 (coupon = 12%)
  • Year-1 and Year-2 short rates
    » $ 894.78 = d_1 \times 30 + d_2 \times 1,030
    » $ 1,053.87 = d_1 \times 120 + d_2 \times 1,120
  • Solve the system: $d_2 = 0.8417$, $d_1 = 0.9259$
  • Conclude ...

Measurements with Coupon Bonds 3

• Example (continued)

\[
r_1 = \frac{1}{d_1} - 1 = \frac{1}{0.9259} - 1 \Rightarrow r_1 = 8\%
\]

\[
r_2 = \frac{1}{(1 + r_1) \times d_2} - 1 = \frac{1}{(1 + 8\%) \times 0.8417} - 1 \Rightarrow r_2 = 10\%
\]
Measurements with Coupon Bonds 4

- Practical problems
  - pricing errors
  - taxes
    » are investors homogenous?
  - investors can sell bonds prior to maturity
  - bonds can be called, put or converted
  - prices quotes can be stale
    » market liquidity

- Estimation
  - statistical approach

Forward Rate Agreements

- What
  - contracts between 2 parties
to lock in forward interest rates

- How?
  - cash-settled contract
    » payment = interest cost change
    » on a nominal (or notional) sum of money
    » if interest rate at that time ≠ agreed-upon interest rate
  - seller pays the buyer if interest rate goes up
  - buyer pays the seller if interest rate goes down
Forward Rate Agreements 2

• Amount to be paid

\[
\text{amount paid by the FRA seller} = \left(\text{nominal amount of contract}\right) \times \frac{(S-A) \times (\# \text{ days the FRA runs})}{(\# \text{ days in the year})} \times \frac{1 + S \times (\# \text{ days the FRA runs})}{(\# \text{ days in the year})}
\]

• Hedger
  • By selling an FRA, can lock in interest on deposit
  • By buying an FRA, can lock in cost of loan

• Example (Handout & PS#3)
  • finding quotes & valuing FRA’s
  • Trading FRAs (arbitraging vs. return maximization)

Interest-Rate Derivatives (Recap. slide)

• Forward rate agreements (FRA)
  • OTC contract; users "lock in" implied forward rate

• Interest Rate Futures (IRF): ED & T-Bill Futures
  • exchange traded futures contracts
  • underlying: 90-day interest rate (contrast with FRA)

• Interest-rate Swaps
  • OTC contract; converts exposure: fixed <-> floating
  • Bundle of “time against time +6 months” FRA’s

• Government bonds futures
  • Exchange-traded futures on a long-term government bond
T-Bill & Eurodollar Futures

- Money-market instruments
  - zero-coupon bonds
  - quotes vs. actual yields
- vs. Long-term bonds
  - quotes
    - T-notes and T-bonds
    - corporate bonds
  - accrued interest

Short-term Bond Prices & Yield Quotes

- T-bills
  - sold at discount to par (typ. $10,000; minimum is $1,000)
    - “capital gain” treated as interest; federal tax only
  - primary market: U.S. Treasury auctions
    - weekly (Mondays; maturity = mostly 91 or 182 days)
    - formerly: every trimester (52 weeks)
  - secondary market
- Other short-term instruments
  - same conventions for quotes (similar idea for futures)
Short-term Bond Prices & Yield Quotes 2

• Yields on T-Bills

  – bank discount yield: \[ \frac{\text{Par} - \text{Price}}{\text{Par}} \times \frac{360}{n} \]
    » used for futures

  – bond equivalent yield: \[ \frac{\text{Par} - \text{Price}}{\text{Price}} \times \frac{365}{n} \]

  – effective annual yield: \[ \frac{\text{Par}^{\frac{365}{n}}}{\text{Price}} - 1 \]

Short-term Bond Prices & Yield Quotes 3

• BDY example

  • a 60-day T-bill has a BDY of 6.81% \((based on ask)\)
    » in the newspaper, the bill would be quoted at
      \[ 100(1 - 0.0681/6) = 98.865 \]
      » the bill’s ask price would be
      \[ = 10,000 \times [100\% - (6.81\% / 6)] \]
      \[ = 9,886.50 \]
    » the bill’s effective annual yield would be
      \[ \text{EAY} = (10,000 / 9,886.50)^{(365/60)} - 1 \]
      \[ = 7.19\% \]
Eurodollar Futures

• What?
  • futures contract on 3-month, $1m eurodeposit
    » underlying = hypothetical deposit “made” at LIBOR,
      starting 3rd Wed. of delivery month
  • traded on CME/SIMEX, cash-settled
  • maturities up to 10 years into the future

• Our discussion
  • market microstructure
  • futures rate
    » vs. forward rate
    » pricing: theory, empirics and practice

Eurodollar Futures 2

• Market microstructure
  • contracts available
    – long maturities (up to 10 years)
      » M-J-S-D
    – short maturities
      » more months
  • settlement
    – in cash
    – 3rd Wednesday of delivery month (why?)
    – last mark-to-market rate is 90-day LIBOR, settlement day
  • underlying variable = 3-month Libor at settlement
Eurodollar Futures 3

• Pricing example
  • Quotes
    - Z = index value = 100 - (annualized) futures deposit rate
    - contract value
      = $10,000 [100 - 0.25(1-Z)] = $1,000,000 [1 - 0.25 (100-Z)\%]
  • Example: June 2003 futures; Z= 95.53
    - annualized futures deposit rate = (100-95.53)\% = 4.47\%
    - contract value = $1,000,000[(100-0.25 (100-95.53))\%] = $988,825
      » 1 b.p. change ↵ $25 change in contract value
  • final marking to market
    - on expiration day, futures price = 100-R
      » R = 90-day Libor (quarterly basis and actual/360 day count)

Eurodollar Futures 4

• FRA and IRF as hedging tools
  – FRA
    • seller (short) pays the buyer if interest rate goes up
      » so: seller locks in the interest return on a deposit
      » i.e: seller gets fixed rate (and pays variable rate)
    • buyer (long) pays the seller if interest rate goes down
      » so: buyer locks in the cost of a loan
      » i.e.: buyer gets variable rate (and pays fixed rate)
  – IRF: just the opposite
    » long (“buyer”) locks in the interest return on a deposit
    » short (“seller”) locks in the cost of a loan
Eurodollar Futures 5

- FRA and IRF as hedging tools (*continued*)
  - FRA
    - if you want to hedge against rates’ going up,
      then, buy an FRA
      » buyer locks in cost of loan
      » i.e., hedged buyer pays fixed rate
  - IRF: just the opposite
    - if you want to hedge against rates’ going up,
      then “*keep your shorts on*”
      » i.e., sell an IRF

Eurodollar Futures 6

- Futures rate *vs.* forward rate
  - theory
    - forward rate < futures rate (*why?*)
  - empirically
    - short maturity
      » not much of a difference
    - long maturity
      » much larger difference
  - practice (---/---)
Eurodollar Futures 7

• Futures rate vs. forward rate
  – (---/---) in practice
    • assume interest rates are continuously compounded
    • forward rate = futures rate - (1/2) \( \sigma^2 t_1 t_2 \)
      – where
        » \( \sigma \) = annual % std deviation of LIBOR
        » \( t_1 \) = contract delivery (in years)
        » \( t_2 \) = end of delivered eurodeposit (in years)
  • numerical example?

T-Bill Futures (NOT Exam Material)

• Basic idea
  • similar to eurodollar futures (size, dates, etc.)
  • Differences:
    – underlying variable = 13-week (3-mo) T-Bill at settlement
    – Tick = \( \frac{1}{2} \) point (vs. 1 pt for Eurodollar futures)
    – Not traded since 2003! Only of historical interest

• Quotes
  • \( Z \leftrightarrow \) index; contract value = $1m [1 - 0.25 (100-Z)\%]
  • Example:
    – 1.25% T-bill disc. rate for delivery month \( \rightarrow Z=98.75 \)
T-Bill Futures 2

• Market microstructure
  • contracts available at the CME
    – maturities
      » M-J-S-D
    – nominal value: $1 million

• settlement
  – in cash
  – 3rd Wednesday of delivery month
  – last mark-to-market rate is T-bill rate, settlement day
    » highest discount rate accepted in U.S. Treasury’s 91-day
      T-bill auction in week of 3rd Wed. of contract month