Derivatives & Risk Management

• So far
  – Forwards & futures
  – Swaps

• This lecture set
  – Hedging with futures

Part IV: Hedging with Futures
Arguments in Favor of Hedging

1. Companies should
   + focus on the main business they are in
   + take steps to minimize risks arising from interest rates, exchange rates, and other market variables

2. Hedging increases firm value
   *mixed empirical evidence*
Arguments against Hedging

• Shareholders are usually well diversified and can make their own hedging decisions
  – So, why hedge?
• It may increase risk to hedge when competitors do not
  – Why?
• Who wants to explain to shareholders/boss when there is a loss on the hedge and a gain on the underlying can be difficult

Long & Short Hedges

• A long futures hedge is appropriate when you know you will purchase an asset in the future and want to lock in the price

• A short futures hedge is appropriate when you know you will sell an asset in the future & want to lock in the price
Hedging with Futures

- Forward hedge: customized, no collateral (caveat)
- Futures hedge:
  - Margin calls $\rightarrow$ cash-flow risk
  - Relies on standard contract $\rightarrow$ basis risk
    - Maturity mismatch
    - Commodity mismatch
  - Roll-over risk
    - Maturity mismatch

1. Basis Risk

- Basis
  - difference between spot & futures prices

- Basis risk
  - arises because of the uncertainty about the basis when the hedge is closed out
Long Hedge

- Suppose that
  \[ F_1 : \text{Initial Futures Price} \]
  \[ F_2 : \text{Final Futures Price} \]
  \[ S_2 : \text{Final Asset Price} \]
- You hedge the future purchase of an asset by entering into a long futures position
- Cost of Asset = \[ S_2 - (F_2 - F_1) = F_1 + \text{Basis} \]
  - *No uncertainty if you close out on delivery day*

Short Hedge

- Suppose again that
  \[ F_1 : \text{Initial Futures Price} \]
  \[ F_2 : \text{Final Futures Price} \]
  \[ S_2 : \text{Final Asset Price} \]
- You hedge the future sale of an asset by entering into a short futures position
- Price realized = \[ S_2 + (F_1 - F_2) = F_1 + \text{Basis} \]
Convergence of Futures to Spot
(Hedge initiated at time $t_1$ and closed out at time $t_2$)
(Hedge based on “own” futures maturing at $T$)

<table>
<thead>
<tr>
<th>Time</th>
<th>Spot Price</th>
<th>Futures Price</th>
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<td>$t_1$</td>
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<td>$t_2$</td>
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Buy or sell the underlying at $S_2$

“Lock” at $F_1$

Exit hedge at $F_2$

The spot price increased from $t_1$ to $t_2$, but the futures price also went up

Convergence of Futures to Spot
(Hedge initiated at time $t_1$ and closed out at time $t_3$)
(Hedge based on “own” futures maturing at $T$)

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Buy or sell the underlying at $S_3$

“Lock” at $F_1$

Exit hedge at $F_3$

Not so good – futures price dropped a lot from $t_1$ to $t_3$, but spot price is the same at $t_3$ as it was at $t_1$. 
Choice of Contract

- **Maturity:** Choose delivery month as close as possible to, but later than, hedge horizon
  - *Alternative:* “earlier than”?
- **Underlying:** When there is no futures on the asset being hedged, choose the contract whose futures price is most highly correlated with the asset price. This is known as cross hedging.
  - *Liquidity?* sometimes, better to use a cross-hedge with liquid contract than an illiquid own hedge.

Optimal Hedge Ratio

Proportion of the exposure that should optimally be hedged is

\[ \rho \frac{\sigma_s}{\sigma_F} \]

where

- \( \sigma_s \) is the standard deviation of \( \Delta S \), the change in the spot price during the hedging period,
- \( \sigma_F \) is the standard deviation of \( \Delta F \), the change in the futures price during the hedging period
- \( \rho \) is the coefficient of correlation between \( \Delta S \) and \( \Delta F \).
2. Rolling Hedges Forward

- Stack-and-roll: use a series of short-dated futures to increase the life of a hedge
  - *Implicit assumption*: price change for stack = average price change for term structure

- Each time we switch from 1 futures contract to another we incur a type of basis risk

- Examples:
  - MetallGesellschaft; GSCI roll

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Futures & Portfolio Management

- Why?
  - 1. Hedging (*= getting beta down to 0*)
    - Desire to be out of the market for a short period of time.
      » Hedging may be cheaper than selling the portfolio and buying it back.
    - Desire to hedge systematic risk
      » Appropriate when you feel that you have picked stocks that will outperform the market.
      » -> “portable alpha”
  - 2. Changing portfolio beta

- How? Optimal hedge ratio
1. Hedging Using Index Futures

To hedge the risk in a portfolio the number of contracts that should be shorted is

$$\beta \frac{P}{A}$$

where

- $P$ is the value of the portfolio,
- $\beta$ is its beta, and
- $A$ is the value of the assets underlying one futures contract.

Example: Getting beta to 0

Value of S&P 500 is 1,000
Value of Portfolio is $5 million
Beta of portfolio is 1.5

What position in futures contracts on the S&P 500 is necessary to hedge the portfolio?

- value of S&P futures is $250 \times \text{index} = $250,000
- $N^* = 1.5 \times \frac{5m}{250k} = 30$ contracts
2. Changing Beta

- What position is necessary to reduce the beta of the portfolio to 0.75?
  - Shorting 30 contracts hedges fully
  - So shorting 15 contracts cuts the exposure in half

- What position is necessary to increase the beta of the portfolio to 2.0?
  - Go long 10 contracts

Why Hedge an Equity Portfolio?

- Desire to be out of the market for a short period of time.
  - Hedging may be cheaper than selling the portfolio and buying it back.

- Desire to hedge systematic risk
  - Appropriate when you feel that you have picked stocks that will outperform the market.

  ➔ “portable alpha”
Portable alpha

Suppose stocks in your portfolio have an average beta of 1.0, but you feel they have been chosen well and will outperform the market in both good and bad times.

Hedging ensures that the return you earn is the risk-free return plus the excess return of your portfolio over the market ("Portable alpha").

Hedging Price of Individual Stock

- Similar to hedging a portfolio
- Does not work as well because only the systematic risk is hedged
- The unsystematic risk that is unique to the stock is not hedged