Derivatives & Risk Management

• Previous lecture set:
  – Futures vs. forwards
  – Stock Index Futures

• This lecture set – Part III
  – Interest-Rate Derivatives
    • FRAs
    • T-bills futures & Euro$ Futures

Part III:
Interest Rate Derivatives

Derivatives “of Interest”

• Interest-Rate Derivatives
  • Contracts on short-term interest rates
    » FRAs, Eurodollar futures (also, T-bills futures)
    » (Single-currency) Interest-rate (IR) Swaps
  • Futures on long-term interest rates
    » e.g., T-bonds & T-notes futures, Bund futures

• Currency derivatives
  • Forwards and futures on FX; FX swaps
  • Currency swaps (= cross-currency interest-rate swaps)

• Relative importance: ISDA + BIS figures
Forward Interest Rates & FRA’s

- Background
  - bond pricing
  - term structure of interest rates & pure yield curve
  - forward interest rate (aka implied forward short rate)

- Forward rate agreements
  - market microstructure
  - locking in rates with FRA’s

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Bond Pricing

- Equation for a coupon bond:
  - \( P = \text{PV(annuity)} + \text{PV(final payment)} \)
  - \( P = \frac{\text{coupon}}{\text{ytm}} \cdot \frac{\text{Par}}{\sum (1 + \text{ytm})} \)
  - Terminology: \( T = \text{maturity}; ytm = \text{yield to maturity} \)

- Example: \( C_t = 40 \); Par = $1,000; disc. rate = 4%; \( T = 60 \)
  \[
  P = \sum_{t=1}^{60} \frac{40}{(1+0.04)^t} + \frac{1,000}{(1+0.04)^{60}} = 904.94 + 95.06 = 1,000
  \]

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Bond Pricing 2

- Equation for a zero-coupon bond:
  - \( P = \text{PV (final payment)} \)
  - \( P = \frac{\text{Par}}{(1+y)^T} \)
  - Terminology: \( y = \text{T-year spot rate} \)

- Example: \( C_t = 0 \); Par = $1,000; disc. rate = 4%; \( T = 60 \)
  \[
  P = \frac{1,000}{(1+0.04)^{60}} = 95.06
  \]
Bond Pricing 3

- Why focus on zeroes?
  - The \textit{ytm} of coupon bonds is an average of the spot rates of each of the cash flows (idea: reinvestment)

\[
P = \sum_{i=}^{\infty} \frac{\text{Coupon}_i}{(1+y_{tm})^i}, \quad Par = \sum_{i=}^{\infty} \frac{\text{Coupon}_i}{(1+y_{i})^i}
\]

- The \textit{ytm} of zeroes (i.e., the spot rate) is not corrupted by these reinvestment issues

Term Structure of Interest Rates

- Basic question
  - link between spot rates (= \textit{ytm} on zeroes) & maturity

- Bootstrapping short rates from strips
  - forward rates and expected future short rates

- Interpreting the term structure
  - does the term structure contain information?
    - certainty vs. uncertainty

- Recovering short rates from coupon bonds

“Term”inology

- Term structure = yield curve
  - = plot of the YTM as a function of bond maturity
  - Pure yield curve (special case)
    - = plot of the spot rate by time-to-maturity

- Short rate vs. spot rate
  - both are “zero rates”
  - 1-period rate vs. multi-period yield (BKM4 Fig. 14.3)
  - spot rate = current rate appropriate to discount a cash-flow of a given maturity
Extracting Info re: Short Interest Rates

- From zeroes
  - non-linear regression analysis
  - bootstrapping
- From coupon bonds (NOT Exam Material)
  - system of equations
  - regression analysis (no measurement errors)
- Certainty vs. uncertainty
  - forward rate vs. expected future (spot) short rate

(Implied) Forward Interest Rates

- Definition #1
  - forward interest rate
    - for a given period in the future
  - = interest rate
    - implied by current spot rates
- Definition #2
  - “break-even rate”
  - that equates
  - the payoffs of roll-over and LT strategies

Bootstrapping Fwd Rates from Zeroes

- Forward rate
  - “break-even rate”
  - equating the payoffs of ST roll-over vs. LT strategies
  - n years @ y_n vs. (n-1) years @ y_{n-1} plus one year at f_n
  \[ n\times y_n = (n-1)\times y_{n-1} + 1\times f_n \]
- Intuitive formula
  - \( f_1 = y_1 \) and \( f_n = n\times y_n - (n-1)\times y_{n-1} \)
Bootstrapping Fwd Rates from Zeroes 2

- Forward rate
  - “break-even rate”
  - equating the payoffs of ST roll-over vs. LT strategies
  - \( n \) years @ \( y_n \) vs. \((n-1)\) years @ \( y_{n-1} \) plus one year at \( f_n \)
    \[ (1 + y_n)^n = (1 + y_{n-1})^{n-1}(1 + f_n) \]

- Precise formula
  \[ f_1 = y_1 \quad \text{and} \quad f_n = \frac{(1 + YTM_{n-1})^n}{(1 + YTM_n)^{n-1}} - 1 \]

Bootstrapping Fwd Rates from Zeroes 3

- Example 1:
  - BKM4 Table 14.2 & Fig.14.1; BKM9 T15.1 & Fig.15.3
  - 4 bonds, all zeroes (reimbursable at par of $1,000)
    | T (maturity) | Price  | YTM (spot rate) |
    |-------------|--------|-----------------|
    | 1           | $925.93| 8%              |
    | 2           | $841.75| 8.995%          |
    | 3           | $758.33| 9.66%           |
    | 4           | $683.18| 9.993%          |

Bootstrapping Fwd Rates from Zeroes 4

- Forward interest rate for year 1
  \[ \frac{\$925.93}{1 + (1 + f_1)} = \frac{\$1,000}{1 + y_1} \Rightarrow f_1 = y_1 = 8\% \]

- Forward interest rate for year 2
  \[ \frac{\$841.75}{1 + (1 + f_2)(1 + f_3)} = \frac{\$1,000}{1 + 8\%(1 + f_3)} \Rightarrow f_2 = 10\% \]
  \[ \frac{\$841.75}{1 + f_2} = \frac{\$925.93}{1 + f_3} \Rightarrow f_3 = 10\% \]
Bootstrapping Fwd Rates from Zeroes 5

- Forward rates for years 3 and 4
  - keep applying the method
  - you find \( f_3 = 11\% = f_4 \)

- General Formula
  \[
  f_i = \frac{(1+YTM_i)^n}{(1+YTM_{i-1})^n-1}
  \]

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Bootstrapping Fwd Rates from Zeroes 6

Example 2: Intuitive ("quick & dirty") forward rates

<table>
<thead>
<tr>
<th>Zero-Coupon Rates</th>
<th>Bond Maturity</th>
<th>(1yr) Fwd Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_1 = 12.00% )</td>
<td>1</td>
<td>( f_1 = y_1 = 12% )</td>
</tr>
<tr>
<td>( y_2 = 11.75% )</td>
<td>2</td>
<td>( f_2 = 11.5% )</td>
</tr>
<tr>
<td>( y_3 = 11.25% )</td>
<td>3</td>
<td>( f_3 = 10.25%^* )</td>
</tr>
<tr>
<td>( y_4 = 10.00% )</td>
<td>4</td>
<td>( f_4 \approx 6.25%^* )</td>
</tr>
<tr>
<td>( y_5 = 9.25% )</td>
<td>5</td>
<td>( f_5 \approx 6.25%^* )</td>
</tr>
</tbody>
</table>

* If computed exactly, \( f_3 = 10.26\%; f_4 = 6.33\%; f_5 = 6.30\% \) (we’ll show this below)

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Bootstrapping Fwd Rates from Zeroes 7

Example 2: "Formal" forward rates

<table>
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<td>5</td>
<td>( f_5 \approx 6.30%^* )</td>
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* If computed quickly, \( f_3 = 10.25\%; f_4 = 6.25\%; f_5 = 6.25\% \)
Fwd Rate & Expected Future Short Rate

- **Q:** Does IFR equal expected short? (is $f_t = r_t$?)
- **A:** Interpreting the yield curve under uncertainty
  - **Short perspective** *(often observed $\Rightarrow$ exam material)*
    - liquidity preference theory (investors)
    - liquidity premium theory (issuer)
  - **Others:** NOT Exam Material
    - Expectations hypothesis
    - Long perspective
    - Market Segmentation vs. Preferred Habitat

Fwd Rate & Exp. Future Short Rate 2

- **Short perspective**
  - liquidity preference theory (“short” investors)
    - investors need to be induced to buy LT securities
    - example: 1-year zero at 8% vs. 2-year zero at 8.995%
  - liquidity premium theory (issuer)
    - issuers prefer to lock in interest rates
  - $f_2 \geq E[r_2]$
  - $f_2 = E[r_2] + \text{liquidity (or risk) premium}$

Fwd Rate & Exp. Future Short Rate 3

- **Long perspective** *(NOT Exam Material)*
  - “long investors” wish to lock in rates
    - roll over a 1-year zero at 8%
    - or lock in via a 2-year zero at 8.995%
  - $E[r_2] = f_2$
  - $f_2 = E[r_2] - \text{liquidity (or risk) “premium”}$
Fwd Rate & Exp. Future Short Rate 4

• Expectation Hypothesis (NOT Exam Material)
  • risk premium = 0 and \( E[r_2] = f_2 \)
  • idea: “arbitrage”

• Market segmentation theory (NOT Exam Material)
  • idea: clienteles
    » ST and LT bonds are not substitutes
  • reasonable?

• Preferred Habitat Theory (NOT Exam Material)
  • investors do prefer some maturities
  • temptations exist

Fwd Rate & Exp. Future Short Rate 5

• In practice
  • liquidity preference + preferred habitat
    » hypotheses have the edge

• Example 2 (continued)

Fwd Rate & Exp. Future Short Rate 6

• Example 2: “Quick & dirty” forward rates

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</table>

\(^*\) If computed exactly, \( f_3 = 10.26\%; f_4 = 6.33\%; f_5 = 6.30\% \) (we’ll show this below)
Fwd Rate & Exp. Future Short Rate 7

Example 2: “Quick” expected future short rates

<table>
<thead>
<tr>
<th>Period</th>
<th>(1yr) Fwd Rate</th>
<th>Expected short rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>f_1 = y_1 = 12%</td>
<td>N.A</td>
</tr>
<tr>
<td>2</td>
<td>f_2 = 11.5%</td>
<td>E(y_1') = r_2 = 11%</td>
</tr>
<tr>
<td>3</td>
<td>f_3 = 10.25%</td>
<td>E(y_1'') = r_3 = 9.75%</td>
</tr>
<tr>
<td>4</td>
<td>f_4 = 6.25%</td>
<td>E(y_1''') = r_4 = 5.75%</td>
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<td>5</td>
<td>f_5 = 6.25%</td>
<td>E(y_1''''') = r_5 = 5.75%</td>
</tr>
</tbody>
</table>

*: Assumes a constant 0.5% per year liquidity premium

Fwd Rate & Exp. Future Short Rate 8

LT rates aggregate exp’d short rates + LP

Example 3:
- short term rates: r_1 = r_2 = r_3 = 10%
- liquidity premium = constant 1% per year

\[ y_1 = r_1 = 10\% \]
\[ y_2 = \sqrt{(1 + r_1)(1 + f_2)} - 1 = \sqrt{(1+10\%)(1+10\%+1\%)} - 1 = 10.5\% \]
\[ y_3 = \frac{1}{2}(1 + r_1)(1 + f_2)(1 + f_3) - 1 = \frac{1}{2}(1+10\%)(1+11\%)(1+11\%) - 1 = 10.67\% \]

Measurement: Zeroes vs. Coupon Bonds

- Zeroes
  - ideal
  - lack of data may exist (need zeroes for all maturities)

- Coupon Bonds (Next 4 pages NOT Exam Material)
  - plentiful
  - coupons and their reinvestment
    - low coupon rate vs. high coupon rate
    - short term rates → they may have different YTM
Measurements with Coupon Bonds

• Example
  • short rates are 8% & 11% for years 1 & 2; certainty
  • 2-year bonds; Par = $1,000; coupon = 3% or 12%
  • Bond 1:
    \[ \frac{\$30}{(1+8\%)} + \frac{\$1,030}{(1+8\%)(1+11\%)} = \$894.78 \Rightarrow YTM = 8.98\% \]
  • Bond 2:
    \[ \frac{\$120}{(1+8\%)} + \frac{\$1,120}{(1+8\%)(1+11\%)} = \$1,053.87 \Rightarrow YTM = 8.94\% \]

Measurements with Coupon Bonds 2

• Example
  • 2-year bonds; Par = $1,000; coupon = 3% or 12%
  • Prices: $894.78 (coupon = 3%); $1,053.87 (coupon = 12%)
  • Year-1 and Year-2 short rates
    \[ \begin{align*}
    &\text{\$894.78} = d_1 \times 30 + d_2 \times 1,030 \\
    &\text{\$1,053.87} = d_1 \times 120 + d_2 \times 1,120
    \end{align*} \]
  • Solve the system: \( d_2 = 0.8417, d_1 = 0.9259 \)
  • Conclude ...

Measurements with Coupon Bonds 3

• Example (continued)
  \[ r_1 = \frac{1}{d_1} - 1 = \frac{1}{0.9259} - 1 \Rightarrow r_1 = 8\% \]
  \[ r_2 = \frac{1}{(1 + r_1) \times d_2} - 1 = \frac{1}{(1 + 8\%) \times 0.8417} - 1 \Rightarrow r_2 = 10\% \]
Measurements with Coupon Bonds

- **Practical problems**
  - pricing errors
  - taxes
    - are investors homogenous?
  - investors can sell bonds prior to maturity
  - bonds can be called, put or converted
  - prices quotes can be stale
    - market liquidity

- **Estimation**
  - statistical approach

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Forward Rate Agreements

- **What**
  - contracts between 2 parties
to lock in forward interest rates

- **How?**
  - cash-settled contract
    - payment = interest cost change
    - on a nominal (or notional) sum of money
    - if interest rate at that time ≠ agreed-upon interest rate
      - **seller pays** the buyer if interest rate goes up
      - **buyer pays** the seller if interest rate goes down

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Forward Rate Agreements 2

- **Amount to be paid**

\[
\text{amount paid by the FRA seller} = (\text{nominal amount of contract}) \times (1 + S) \times \left(\frac{\text{days the FRA runs}}{\text{days in the year}}\right)\]

- **Hedger**
  - By selling an FRA, can lock in interest on deposit
  - By buying an FRA, can lock in cost of loan

- **Example** (*Handout & PS#3*)
  - finding quotes & valuing FRA’s
  - Trading FRAs (arbitraging vs. return maximization)
Interest-Rate Derivatives *(Recap. slide)*

- Forward rate agreements (FRA)
  - OTC contract; users “lock in” implied forward rate
- Interest Rate Futures (IRF): ED & T-Bill Futures
  - Exchange traded futures contracts
  - Underlying: 90-day interest rate *(contrast with FRA)*
- Interest-rate Swaps
  - OTC contract; converts exposure: fixed $\rightarrow$ floating
  - Bundle of “time against time +6 months” FRA’s
- Government bonds futures
  - Exchange-traded futures on a long-term government bond

T-Bill & Eurodollar Futures

- Money-market instruments
  - Zero-coupon bonds
  - Quotes vs. actual yields
- vs. Long-term bonds
  - Quotes
    - T-notes and T-bonds
    - Corporate bonds
  - Accrued interest

Short-term Bond Prices & Yield Quotes

- T-bills
  - Sold at discount to par (typ. $10,000; minimum is $1,000)
    - “capital gain” treated as interest; federal tax only
  - Primary market: U.S. Treasury auctions
    - Weekly (Mondays; maturity = mostly 91 or 182 days)
    - Formerly: every trimester (52 weeks)
  - Secondary market
- Other short-term instruments
  - Same conventions for quotes *(similar idea for futures)*
Short-term Bond Prices & Yield Quotes 2

- Yields on T-Bills
  - bank discount yield: \( \frac{\text{Par} - \text{Price}}{\text{Par}} \times \frac{360}{n} \)
    - used for futures
  - bond equivalent yield: \( \frac{\text{Par} - \text{Price}}{\text{Price}} \times \frac{365}{n} \)
  - effective annual yield: \( \frac{\text{Par}^{\frac{365}{n}}}{\text{Price}} - 1 \)

Short-term Bond Prices & Yield Quotes 3

- BDY example
  - a 60-day T-bill has a BDY of 6.81% (based on ask)
    - in the newspaper, the bill would be quoted at
      \[ 100(1 - 0.0681/6) = 98.865 \]
    - the bill’s ask price would be
      \[ = $10,000 \times [100\% - (6.81\% / 6)] \]
      \[ = $9,886.50 \]
    - the bill’s effective annual yield would be
      \[ \text{EAY} = \left(\frac{10,000}{9,886.50}\right)^{\frac{365}{60}} - 1 \]
      \[ = 7.19\% \]

Eurodollar Futures

- What?
  - futures contract on 3-month, $1m eurodeposit
    - underlying = hypothetical deposit “made” at LIBOR, starting 3rd Wed. of delivery month
  - traded on CME/SIMEX, cash-settled
  - maturities up to 10 years into the future

- Our discussion
  - market microstructure
  - futures rate
    - vs. forward rate
    - pricing: theory, empirics and practice
Eurodollar Futures 2

- Market microstructure
  - contracts available
    - long maturities (up to 10 years)
      » M-J-S-D
    - short maturities
      » more months
  - settlement
    - in cash
    - 3rd Wednesday of delivery month (why?)
    - last mark-to-market rate is 90-day LIBOR, settlement day
  - underlying variable = 3-month Libor at settlement

Eurodollar Futures 3

- Pricing example
  - Quotes
    - \( Z = \) index value = 100 - (annualized) futures deposit rate
    - contract value
      \[ = 10,000 \times (100 - 0.25(1 - Z)) = 1,000,000 \times (1 - 0.25(100 - Z)\%) \]
  - Example: June 2003 futures; \( Z = 95.53 \)
    - annualized futures deposit rate \( = (100 - 95.53)\% = 4.47\% \)
    - contract value
      \[ = 1,000,000 \times 100 \times 0.25 \times 0.47 = 988,825 \]
      » 1 b.p. change => $25 change in contract value
  - final marking to market
    - on expiration day, futures price \( = 100 - R \)
      » \( R = 90\)-day Libor (quarterly basis and actual/360 day count)

Eurodollar Futures 4

- FRA and IRF as hedging tools
  - FRA
    - seller (short) pays the buyer if interest rate goes up
      » so: seller locks in the interest return on a deposit
      » i.e.: seller gets fixed rate (and pays variable rate)
    - buyer (long) pays the seller if interest rate goes down
      » so: buyer locks in the cost of a loan
      » i.e.: buyer gets variable rate (and pays fixed rate)
  - IRF: just the opposite
    » long ("buyer") locks in the interest return on a deposit
    » short ("seller") locks in the cost of a loan
Eurodollar Futures 5

- FRA and IRF as hedging tools (continued)
  - FRA
    - if you want to hedge against rates’ going up, then, buy an FRA
      » buyer locks in cost of loan
      » i.e., hedged buyer pays fixed rate
  - IRF: just the opposite
    - if you want to hedge against rates’ going up, then “keep your shorts on”
      » i.e., sell an IRF

Eurodollar Futures 6

- Futures rate vs. forward rate
  - theory
    - forward rate < futures rate (why?)
  - empirically
    - short maturity
      » not much of a difference
    - long maturity
      » much larger difference
  - practice (---/---)

Eurodollar Futures 7

- Futures rate vs. forward rate
  - (---/---) in practice
    - assume interest rates are continuously compounded
    - forward rate = futures rate - (1/2) $\sigma^2 t_1 t_2$
      - where
        » $\sigma$ = annual % std deviation of LIBOR
        » $t_1$ = contract delivery (in years)
        » $t_2$ = end of delivered eurodeposit (in years)
    - numerical example?
T-Bill Futures (NOT Exam Material)

• Basic idea
  • similar to eurodollar futures (size, dates, etc.)
  • Differences:
    – underlying variable = 13-week (3-mo) T-Bill at settlement
    – Tick = \( \frac{1}{2} \) point (vs. 1 pt for Eurodollar futures)
    – Not traded since 2003! Only of historical interest

• Quotes
  • \( Z \to \) index; contract value = $1m \( \left[ 1 - 0.25 \times (100-Z)\% \right] \)
  • Example:
    – 1.25% T-bill discount rate for delivery month \( \Rightarrow Z=98.75 \)

T-Bill Futures 2

• Market microstructure
  • contracts available at the CME
    – maturities
      » M-J-S-D
    – nominal value: $1 million
  • settlement
    – in cash
    – 3rd Wednesday of delivery month
    – last mark-to-market rate is T-bill rate, settlement day
      » highest discount rate accepted in U.S. Treasury’s 91-day T-bill auction in week of 3rd Wed. of contract month