Derivatives & Risk Management

- Previous lecture set:
  - Futures vs. forwards
  - Stock Index Futures

- This lecture set – Part III
  - Interest-Rate Derivatives
    - FRAs
    - T-bills futures & Euro$ Futures

Part III:
Interest Rate Derivatives

Derivatives “of Interest”

- Interest-Rate Derivatives
  - Contracts on short-term interest rates
    - FRAs, Eurodollar futures (also, T-bills futures)
    - (Single-currency) Interest-rate (IR) Swaps
  - Futures on long-term interest rates
    - e.g., T-bonds & T-notes futures, Bund futures

- Currency derivatives
  - Forwards and futures on FX; FX swaps
  - Currency swaps (= cross-currency interest-rate swaps)

- Relative importance: ISDA + BIS figures

Forward Interest Rates & FRA’s

- Background
  - bond pricing
  - term structure of interest rates & pure yield curve
  - forward interest rate (aka implied forward short rate)

- Forward rate agreements
  - market microstructure
  - locking in rates with FRA’s

Bond Pricing

- Equation for a coupon bond:
  \[ P = \text{PV(annuity)} + \text{PV(final payment)} \]
  \[ = \sum_{i=1}^{T} \frac{\text{coupon}}{(1+y)^i} \cdot \frac{\text{Par}}{(1+y)} \]

  - Terminology: \( T \) = maturity; \( y \) = yield to maturity

- Example: \( C_i = \$40; \text{Par} = \$1,000; \text{disc. rate} = 4%; T=60 \)
  \[ P = \sum_{i=1}^{60} \frac{\$40}{(1+0.04)^i} + \frac{\$1,000}{(1+0.04)^{60}} = \$994.94 + \$95.06 = \$1,000 \]

Bond Pricing 2

- Equation for a zero-coupon bond:
  \[ P = \text{PV (final payment)} \]
  \[ = \frac{\text{Par}}{1+y} \]

  - Terminology: \( y \) = T-year spot rate

- Example: \( C_i = \$0; \text{Par} = \$1,000; \text{disc. rate} = 4%; T=60 \)
  \[ P = \frac{\$1,000}{(1+0.04)^{60}} = \$95.06 \]
Bond Pricing 3

- Why focus on zeroes?
  - The *ytm* of coupon bonds is an average of the spot rates of each of the cash flows (idea: reinvestment)
  \[ P = \frac{\sum_{i=1}^{n} \text{Coupon}_i}{\sum_{i=1}^{n} (1+y_{tm})^i} \]
  \[ \text{Par} = \frac{\sum_{i=1}^{n} \text{Coupon}_i}{\sum_{i=1}^{n} (1+y_{tm})^i} \]
  - The *ytm* of zeroes (i.e., the spot rate) is not corrupted by these reinvestment issues

Term Structure of Interest Rates

- Basic question
  - link between spot rates (= *ytm* on zeroes) & maturity

- Bootstrapping short rates from strips
  - forward rates and expected future short rates

- Interpreting the term structure
  - does the term structure contain information?
  - certainty vs. uncertainty

- Recovering short rates from coupon bonds

“Term”inology

- Term structure = yield curve
  - = plot of the *ytm* as a function of bond maturity
  - Pure yield curve (special case)
    - = plot of the spot rate by time-to-maturity

- Short rate vs. spot rate
  - both are “zero rates”
  - 1-period rate vs. multi-period yield (BKM4 Fig. 14.3)
  - spot rate = current rate appropriate to discount a cash-flow of a given maturity

Extracting Info re: Short Interest Rates

- From zeroes
  - non-linear regression analysis
  - bootstrapping

- From coupon bonds (NOT Exam Material)
  - system of equations
  - regression analysis (no measurement errors)

- Certainty vs. uncertainty
  - forward rate vs. expected future (spot) short rate

(Implied) Forward Interest Rates

- Definition #1
  - forward interest rate for a given period in the future
  - = interest rate implied by current spot rates

- Definition #2
  - “break-even rate” that equates the payoffs of roll-over and LT strategies

Bootstrapping Fwd Rates from Zeroes

- Forward rate
  - “break-even rate” equating the payoffs of ST roll-over vs. LT strategies
    \[ n \times y_n = (n-1) \times y_{n-1} + 1 \times f_n \]

- Intuitive formula
  - \[ f_1 = y_1 \]
  - \[ f_n = n \times y_n - (n-1) \times y_{n+1} \]
Bootstrapping Fwd Rates from Zeroes 2

- **Forward rate**
  - "break-even rate"
  - equating the payoffs of ST roll-over vs. LT strategies
  - \( n \) years @ \( y_n \) vs. \((n-1)\) years @ \( y_{n-1} \) plus one year at \( f_n \)
  - \((1+y_n)^n = (1+y_{n-1})^{n-1}(1+f_n)\)

- **Precise formula**
  - \( f_1 = y_1 \) and \( f_n = \frac{(1+YTM_n)^n}{(1+YTM_{n-1})^{n-1}} - 1 \)

Bootstrapping Fwd Rates from Zeroes 3

- **Example 1:**
  - BKM4 Table 14.2 & Fig.14.1; BKM9 T15.1 & Fig.15.3
  - 4 bonds, all zeroes (reimbursable at par of $1,000)

<table>
<thead>
<tr>
<th>( T ) (maturity)</th>
<th>Price</th>
<th>YTM (spot rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$925.93</td>
<td>8%</td>
</tr>
<tr>
<td>2</td>
<td>$841.75</td>
<td>8.995%</td>
</tr>
<tr>
<td>3</td>
<td>$758.33</td>
<td>9.66%</td>
</tr>
<tr>
<td>4</td>
<td>$683.18</td>
<td>9.993%</td>
</tr>
</tbody>
</table>

Bootstrapping Fwd Rates from Zeroes 4

- **Forward interest rate for year 1**
  - $925.93 = \frac{$1,000}{(1+f_1)} \Rightarrow f_1 = y_1 = 8%$

- **Forward interest rate for year 2**
  - $841.75 = \frac{$1,000}{(1+f_1)(1+f_2)} = \frac{\frac{$1,000}{(1+y_1)(1+f_1)}}{(1+y_1)} = \frac{$925.93}{(1+y_1)}$

  $841.75 = \frac{$925.93}{(1+y_1)} \Rightarrow f_2 = 10%$

Bootstrapping Fwd Rates from Zeroes 5

- **Example 2:**
  - Intuitive ("quick & dirty") forward rates

<table>
<thead>
<tr>
<th>Zero-Coupon Rates</th>
<th>Bond Maturity</th>
<th>(1yr) Fwd Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_1 ) = 12.00%</td>
<td>1</td>
<td>( f_1 = y_1 = 12% )</td>
</tr>
<tr>
<td>( y_2 ) = 11.75%</td>
<td>2</td>
<td>( f_2 = 11.5% )</td>
</tr>
<tr>
<td>( y_3 ) = 11.25%</td>
<td>3</td>
<td>( f_3 = 10.25% )</td>
</tr>
<tr>
<td>( y_4 ) = 10.00%</td>
<td>4</td>
<td>( f_4 = 6.25% )</td>
</tr>
<tr>
<td>( y_5 ) = 9.25%</td>
<td>5</td>
<td>( f_5 = 6.25% )</td>
</tr>
</tbody>
</table>

* If computed exactly, \( f_3 = 10.26% \); \( f_4 = 6.33% \); \( f_5 = 6.30% \) (we’ll show this below)

- **Example 2:**
  - General Formula

  \( f_n = YTM_n \)

  \( 1 + f_n = \frac{(1+YTM_n)^n}{(1+YTM_{n-1})^{n-1}} \)

Bootstrapping Fwd Rates from Zeroes 6

- **Example 2:**
  - "Formal" forward rates

<table>
<thead>
<tr>
<th>Zero-Coupon Rates</th>
<th>Bond Maturity</th>
<th>(1yr) Fwd Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_1 ) = 12.00%</td>
<td>1</td>
<td>( f_1 = y_1 = 12% )</td>
</tr>
<tr>
<td>( y_2 ) = 11.75%</td>
<td>2</td>
<td>( f_2 = 11.5% )</td>
</tr>
<tr>
<td>( y_3 ) = 11.25%</td>
<td>3</td>
<td>( f_3 = 10.26% )</td>
</tr>
<tr>
<td>( y_4 ) = 10.00%</td>
<td>4</td>
<td>( f_4 = 6.33% )</td>
</tr>
<tr>
<td>( y_5 ) = 9.25%</td>
<td>5</td>
<td>( f_5 = 6.30% )</td>
</tr>
</tbody>
</table>

* If computed quickly, \( f_3 = 10.25% \); \( f_4 = 6.25% \); \( f_5 = 6.25% \)

* If computed exactly, \( f_3 = 10.26% \); \( f_4 = 6.33% \); \( f_5 = 6.30% \) (we’ll show this below)
Fwd Rate & Expected Future Short Rate

• **Q:** Does IFR equal expected short? (is $f_t = r_t$?)
• **A:** Interpreting the yield curve under uncertainty
  – Short perspective (often observed $\rightarrow$ exam material)
    – liquidity preference theory (investors)
    – liquidity premium theory (issuer)
  – **Others: NOT Exam Material**
    • Expectations hypothesis
    • Long perspective
    • Market Segmentation vs. Preferred Habitat

Fwd Rate & Exp. Future Short Rate 2

• **Short perspective**
  • liquidity preference theory (“short” investors)
    – investors need to be induced to buy LT securities
  • liquidity premium theory (issuer)
    – issuers prefer to lock in interest rates
  • $f_t \geq E[r_t]$ (or risk) premium

Fwd Rate & Exp. Future Short Rate 3

• **Long perspective (NOT Exam Material)**
  • “long investors” wish to lock in rates
    – roll over a 1-year zero at 8%
    – or lock in via a 2-year zero at 8.995%
  • $E[r_2] \geq f_2$
  • $f_2 = E[r_2]$ - liquidity (or risk) “premium”

Fwd Rate & Exp. Future Short Rate 4

• **Expectation Hypothesis (NOT Exam Material)**
  • risk premium = 0 and $E[r_2] = f_2$
  • idea: “arbitrage”
• Market segmentation theory (NOT Exam Material)
  • idea: clienteles
    – ST and LT bonds are not substitutes
  • reasonable?
• Preferred Habitat Theory (NOT Exam Material)
  • investors do prefer some maturities
  • temptations exist

Fwd Rate & Exp. Future Short Rate 5

• In practice
  • liquidity preference + preferred habitat
    – hypotheses have the edge
• Example 2 (continued)

Fwd Rate & Exp. Future Short Rate 6

• **Example 2:** “Quick & dirty” forward rates
  • Zero-Coupon Rates | Bond Maturity | (1yr) Fwd Rate
  • $y_1 = 12.00\%$  | 1  | $f_1 = y_1 = 12\%$
  • $y_2 = 11.75\%$  | 2  | $f_2 = 11.5\%$
  • $y_3 = 11.25\%$  | 3  | $f_3 = 10.25\%$
  • $y_4 = 10.00\%$  | 4  | $f_4 = 6.25\%$*
  • $y_5 = 9.25\%$  | 5  | $f_5 = 6.25\%$*

* If computed exactly, $f_3 \approx 10.26%; f_4 \approx 6.33%; f_5 \approx 6.30\%$ (we’ll show this below)
Fwd Rate & Exp. Future Short Rate 7

• Example 2: “Quick” expected future short rates

<table>
<thead>
<tr>
<th>Period</th>
<th>(1yr) Fwd Rate</th>
<th>Expected short rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$f_1 = y_1 = 12%$</td>
<td>N.A.</td>
</tr>
<tr>
<td>2</td>
<td>$f_2 = 11.5%$</td>
<td>$E(y_1') = r_2 = 11%$</td>
</tr>
<tr>
<td>3</td>
<td>$f_3 = 10.25%$</td>
<td>$E(y_1''') = r_3 = 9.75%$</td>
</tr>
<tr>
<td>4</td>
<td>$f_4 = 6.25%$</td>
<td>$E(y_1''''') = r_4 = 5.75%$</td>
</tr>
<tr>
<td>5</td>
<td>$f_5 = 6.25%$</td>
<td>$E(y_1''''''') = r_5 = 5.75%$</td>
</tr>
</tbody>
</table>

* Assumes a constant 0.5% per year liquidity premium

Fwd Rate & Exp. Future Short Rate 8

• LT rates aggregate exp’d short rates + LP

• Example 3:

• short term rates: $r_1 = r_2 = r_3 = 10\%$

  • liquidity premium = constant 1% per year

  $y_1 = r_1 = 10\%$

  $y_2 = \sqrt{(1 + r_1)(1 + y_1)} - 1 = \sqrt{(1 + 10\%)(1 + 10\%)} - 1 = 10.5\%$

  $y_3 = \sqrt{(1 + r_2)(1 + y_2)} - 1 = \sqrt{(1 + 10\%)(1 + 11\%)} - 1 = 10.67\%$

Measurement: Zeroes vs. Coupon Bonds

• Zeros
  • ideal
  • lack of data may exist (need zeroes for all maturities)

• Coupon Bonds (Next 4 pages NOT Exam Material)
  • plentiful
  • coupons and their reinvestment
    » low coupon rate vs. high coupon rate
    » short term rates → they may have different YTM

Measurements with Coupon Bonds

• Example

  • short rates are 8% & 11% for years 1 & 2; certainty
  • 2-year bonds; Par = $1,000; coupon = 3% or 12%

  • Bond 1:

    \[
    \frac{\$30}{1 + 8\%} + \frac{\$1030}{(1 + 8\%)(1 + 11\%)} = \$894.78 \Rightarrow YTM = 8.98\% 
    \]

  • Bond 2:

    \[
    \frac{\$120}{1 + 8\%} + \frac{\$1120}{(1 + 8\%)(1 + 11\%)} = \$1,053.87 \Rightarrow YTM = 8.94\% 
    \]

Measurements with Coupon Bonds 2

• Example

  • 2-year bonds; Par = $1,000; coupon = 3% or 12%
  • Prices: $894.78 (coupon = 3%); $1,053.87 (coupon = 12%)
  • Year-1 and Year-2 short rates

    $894.78 = d_1 \times 30 + d_2 \times 1,030$
    $1,053.87 = d_1 \times 1,120 + d_2 \times 1,120$
  • Solve the system: $d_2 = 0.8417, d_1 = 0.9259$
  • Conclude ...

Measurements with Coupon Bonds 3

• Example (continued)

  \[
  r_1 = \frac{1}{d_1} - 1 = \frac{1}{0.9259} - 1 \Rightarrow r_1 = 8\% 
  \]

  \[
  r_2 = \frac{1}{(1 + r_1)xd_2} - 1 = \frac{1}{(1 + 8\%)(0.8417)} - 1 \Rightarrow r_2 = 10\% 
  \]
Measurements with Coupon Bonds 4

• Practical problems
  • pricing errors
  • taxes
    » are investors homogenous?
  • investors can sell bonds prior to maturity
  • bonds can be called, put or converted
  • prices quotes can be stale
    » market liquidity

• Estimation
  • statistical approach

Forward Rate Agreements

• What
  • contracts between 2 parties
    to lock in forward interest rates

• How?
  • cash-settled contract
    » payment = interest cost change
    » on a nominal (or notional) sum of money
    » if interest rate at that time ≠ agreed-upon interest rate
  • seller pays the buyer if interest rate goes up
  • buyer pays the seller if interest rate goes down

Forward Rate Agreements 2

• Amount to be paid
  amount paid by the FRA seller = (nominal amount of contract) x \( (S-A) \times \frac{\text{# days the FRA runs}}{\text{# days in the year}} \)

• Hedger
  • By selling an FRA, can lock in interest on deposit
  • By buying an FRA, can lock in cost of loan

• Example (Handout & PS#3)
  • finding quotes & valuing FRA’s
  • Trading FRAs (arbitraging vs. return maximization)

Interest-Rate Derivatives (Recap. slide)

• Forward rate agreements (FRA)
  • OTC contract; users "lock in" implied forward rate

• Interest Rate Futures (IRF): ED & T-Bill Futures
  • exchange traded futures contracts
  • underlying: 90-day interest rate (contrast with FRA)

• Interest-rate Swaps
  • OTC contract; converts exposure: fixed <-> floating
  • Bundle of "time against time + 6 months" FRA’s

• Government bonds futures
  • Exchange-traded futures on a long-term government bond

T-Bill & Eurodollar Futures

• Money-market instruments
  • zero-coupon bonds
  • quotes vs. actual yields

• vs. Long-term bonds
  • quotes
    » T-notes and T-bonds
    » corporate bonds
  • accrued interest

Short-term Bond Prices & Yield Quotes

• T-bills
  • sold at discount to par (typ. $10,000; minimum is $1,000)
    » "capital gain" treated as interest; federal tax only
  • primary market: U.S. Treasury auctions
    » weekly (Mondays; maturity = mostly 91 or 182 days)
    » formerly: every trimester (52 weeks)
  • secondary market

• Other short-term instruments
  • same conventions for quotes (similar idea for futures)
### Short-term Bond Prices & Yield Quotes 2

- **Yields on T-Bills**
  - **Bank discount yield:** \( \frac{\text{Par} - \text{Price}}{\text{Par}} \times \frac{360}{n} \)
    - used for futures
  - **Bond equivalent yield:** \( \frac{\text{Par} - \text{Price}}{\text{Price}} \times \frac{365}{n} \)
  - **Effective annual yield:** \( \frac{\text{Par}}{\text{Price}}^{\frac{365}{n}} - 1 \)

### Short-term Bond Prices & Yield Quotes 3

- **BDY example**
  - A 60-day T-bill has a BDY of 6.81% *(based on ask)*
    - in the newspaper, the bill would be quoted at
      \[ 100(1 - 0.0681/6) = 98.865 \]
    - the bill’s ask price would be
      \[ 10,000 \times [100\% - (6.81\% / 6)] = 9,886.50 \]
    - the bill’s effective annual yield would be
      \[ \text{EAY} = (10,000 / 9,886.50)^{(365/60)} - 1 = 7.19\% \]

### Eurodollar Futures

- **What?**
  - futures contract on 3-month, $1m eurodeposit
    - underlying = hypothetical deposit "made" at LIBOR, starting 3rd Wed. of delivery month
  - traded on CME/SIMEX, cash-settled
  - maturities up to 10 years *into the future*

- **Our discussion**
  - market microstructure
  - futures rate
    - vs. forward rate
    - pricing: theory, empirics and practice

### Eurodollar Futures 2

- **Market microstructure**
  - contracts available
    - long maturities (up to 10 years)
    - M-J-S-D
    - short maturities
    - more months
  - settlement
    - in cash
    - 3rd Wednesday of delivery month *(why?)*
    - last mark-to-market rate is 90-day LIBOR, settlement day
  - underlying variable = 3-month Libor at settlement

### Eurodollar Futures 3

- **Pricing example**
  - **Quotes**
    - Z = index value (annualized) futures deposit rate
    - contract value
      \[ = 10,000(100 - 0.25\%\text{(1-Z)}) = 1,000,000[1 - 0.25 \times (100-Z)\%] \]
  - Example: June 2003 futures; Z= 95.53
    - annualized futures deposit rate = (100-95.53)\% = 4.47\%
    - contract value = $1,000,000[100 - 0.25 \times (100-95.53)\%] = $988,825
  - 1 b.p. change \( \rightarrow \) $25 change in contract value
  - final marking to market
    - on expiration day, futures price = 100-R
    - R = 90-day Libor (quarterly basis and actual/360 day count)

### Eurodollar Futures 4

- **FRA and IRF as hedging tools**
  - **FRA**
    - seller (short) pays *the buyer* if interest rate goes up
      - so: seller locks in the interest return on a deposit
      - i.e.: seller gets fixed rate *(and pays variable rate)*
    - buyer (long) pays *the seller* if interest rate goes down
      - so buyer locks in the cost of a loan
      - i.e.: buyer gets variable rate *(and pays fixed rate)*
  - **IRF**: just the opposite
    - long *("buyer") locks in the interest return on a deposit
    - short *("seller") locks in the cost of a loan
Eurodollar Futures 5

- FRA and IRF as hedging tools (continued)
  - FRA
    - if you want to hedge against rates’ going up, then, buy an FRA
      » buyer locks in cost of loan
      » i.e., hedged buyer pays fixed rate
  - IRF: just the opposite
    - if you want to hedge against rates’ going up, then “keep your shorts on”
      » i.e., sell an IRF

Eurodollar Futures 6

- Futures rate vs. forward rate
  - theory
    - forward rate < futures rate (why?)
  - empirically
    - short maturity
      » not much of a difference
    - long maturity
      » much larger difference
  - practice (---/---)

Eurodollar Futures 7

- Futures rate vs. forward rate
  - (---/---) in practice
    - assume interest rates are continuously compounded
    - forward rate = futures rate - (1/2) \( \sigma^2 t_1 t_2 \)
      - where
        » \( \sigma \) = annual % std deviation of LIBOR
        » \( t_1 \) = contract delivery (in years)
        » \( t_2 \) = end of delivered eurodeposit (in years)
    - numerical example?

T-Bill Futures (NOT Exam Material)

- Basic idea
  - similar to eurodollar futures (size, dates, etc.)
  - Differences:
    - underlying variable = 13-week (3-mo) T-Bill at settlement
    - Tick = ½ point (vs. 1 pt for Eurodollar futures)
    - Not traded since 2003! Only of historical interest
- Quotes
  - \( Z \leftarrow \text{index} \); contract value = $1m \[1 - 0.25(100-Z)\%\]
  - Example:
    - 1.25% T-bill disc. rate for delivery month \( \rightarrow Z=98.75 \)

T-Bill Futures 2

- Market microstructure
  - contracts available at the CME
    - maturities
      » M-J-S-D
    - nominal value: $1 million
  - settlement
    - in cash
    - 3rd Wednesday of delivery month
    - last mark-to-market rate is T-bill rate, settlement day
      » highest discount rate accepted in U.S. Treasury’s 91-day T-bill auction in week of 3rd Wed. of contract month