Practice Set #1: Forward pricing & hedging.

What to do with this practice set?

To help students with the material, eight practice sets with solutions shall be handed out. These sets contain mostly problems of my own design as well as a few carefully chosen, worked-out end-of-chapter problems from Hull. None of these Practice Sets will be graded: the number of "points" for a question solely indicates its difficulty in terms of the number of minutes needed to provide an answer.

Students are strongly encouraged to try hard to solve the practice sets and to use office hours to discuss any problems they may have doing so. The best self-test for a student of her/his command of the material is whether s/he can handle the questions of the relevant practice sets.

The questions on the mid-term and final exams will cover the material covered in class. Their format, in particular, shall in large part reflect questions such as the numerical exercises solved in class and/or the questions in the practice sets.

Question 1. (5 points)

In Feb.09, gold was trading at $950 per ounce for spot delivery. The lease rate was about 0%.
a. If the 1-month interest rate was 3.6% (LIBOR, annualized) and gold storage costs were 1.2% per year (annualized), what was the 1-month (net) cost of carry? Assume no convenience yield.

b. If a gold brokerage was contemporaneously selling gold at $960 for 1-month forward delivery, was there an arb opportunity? Explain briefly. Assume commissions of $15 per ounce to buy or sell gold, for both forward and spot purchases/sales.

(Hint: What should have been the 1-month forward gold price?)

**Question 2 (15 points)**

Suppose that the nearby WTI sweet crude oil futures contract (i.e., the benchmark contract with the nearest expiration) matures in a month, and that the nearby futures price is $98 per barrel.

(i) Based on this information, what should be the OTC (over-the-counter) price of a barrel of WTI for delivery in a month – i.e., what should the 1-month forward price be?

(ii) If you buy 1,000 barrels of oil OTC, for delivery in a month, what will be your cash-flow today? In a month? Between today and the contract’s delivery date? Assume that this OTC forward contract is commodity-settled.

(iii) Would your answer change if the OTC contract were cash-settled (i.e., if it were an NDF or “non-deliverable forward”)?

(iv) a. Suppose that you are a small independent crude oil refiner and refine about ten thousand barrels a month. You have signed forward contracts committing you to sell your output at fixed prices. What price risk are you facing?

   b. To hedge that risk, you wish to hedge the cost of refining 10,000 barrels of oil that you will be receiving and paying for in a month. Should you go long or short? How would you carry out the hedge in practice?

   c. Suppose once again that you are a crude oil refiner, and wish to hedge the cost of refining 10,000 barrels of oil that you will be receiving and paying for in a month. If you decide to use an OTC contract, does it matter whether you use an NDF or a commodity-settled contract to hedge?

   d. In the same situation, would you prefer using a commodity-settled futures instead of a commodity-settled forward (OTC) contract?

**Question 3 (10 points)**

(i) Consider the crude oil refiner of question 2(iv) but suppose that he has not sold his output forward. What hedging strategy would you recommend? (Hint: crack spreads, anyone?)

(ii) Consider a soybean processor who has neither locked in the cost of his inputs (i.e., the actual grains) has not sold his output (i.e., soybean oil and soybean “meal”) forward. What hedging strategy would you recommend? (Hint: crush spreads, anyone?)
Question 4. (5 points)

The direct spot quote for the Canadian dollar in New York is C$1 = USD 0.76. The 180-day swap rate is –2 pts (“minus two points”).

a. What accounts for the difference between the 2 rates? Explain.
b. In the absence of any other information, can you use the 180-day forward quote to forecast the direct spot quote for the Canadian $ in New York, 6 months from now? Explain briefly.

Question 5. (10 points)

Suppose that you are a trader of JP Morgan allowed to do arbitrage. Annualized six-month LIBOR for the Yen and the U.S. dollar are:

Bid (deposit) – Ask (Borrow) ¥ at 0.5%-0.625%
Bid (deposit) – Ask (Borrow) $ at 5.375%-5.5%

From a phone call to a trader at Daiwa Bank, you learn that Daiwa will let customers buy and sell ¥ spot at 100.00-50 ¥/1$. A trader at Barclays is simultaneously quoting bid and ask 6-month swap rates of –300 points (i.e., he will buy and sell ¥ 6-month forward at 97.00-50 ¥/1$).

a. Can you make money out of these quotes? Explain thoroughly.
b. Suppose you must borrow $1m for JP Morgan. What would be your total borrowing cost (i.e., what is the total number of $ you would pay on your $1m loan)?

(Hint: what are your borrowing choices?)

Bonus Question. (5 points – TBD if exam material)

Suppose that the 3-month interest rate in Denmark is about 3.5%. Meanwhile, the equivalent interest rate in England is about 6.5%. All rates are annualized. What should be the annualized 3-month forward discount or premium at which the Danish krone will sell against the pound?
Practice Set #1: Solutions

Question 1, (5 points)

In Feb. 2009, gold was trading at $950 per ounce for spot delivery. The lease rate was about 0%.

a. If the 1-month interest rate was 3.6% (LIBOR, annualized) and gold storage costs were 1.2% per year (annualized), what was the 1-month (net) cost of carry? Assume no convenience yield.

Answer

Given the lease rate and convenience yield are 0, the (annualized) cost of carry was (3.6%+1.2%) = 4.8%.

b. If a gold brokerage was contemporaneously selling gold at $960 for 1-month forward delivery, was there an arb opportunity? Explain briefly. Assume commissions of $15 per ounce to buy or sell gold, for both forward and spot purchases/sales.

(Hint: What should have been the 1-month forward gold price?)

Answer

Gold should have been trading at $950 (1+3.6%+1.2%)^{1/12} or approximately $953.75. The price of $960 therefore looks like it offers an arbitrage opportunity, until you take brokerage fees into account.

Without commissions, you could borrow $950 to buy gold spot for $950/1oz, sell it forward for 960 and make $6.25 per ounce ( = $960 - $950 (1+3.6%+1.2%)^{1/12} ) profits – even taking into account the cost of interest on borrowed funds and the cost to store the gold for a month.

It would cost you $30 to trade the gold, however ($15 to buy spot and $15 to resell it forward), which would wipe out arb profits.

Question 2 (15 points)

Suppose that the nearby WTI sweet crude oil futures contract (i.e., the benchmark contract with the closest expiration date) matures in a month, and that the nearby futures price is $98 per barrel.

(i) Based on this information, what should be the OTC (over-the-counter) price of a barrel of WTI for delivery in a month – i.e., what should the 1-month forward price be?

Answer:

The price should be $98. As mentioned in class (and as is explained in more details in LN 4&5 and in the last page of the class handout on futures marking to market), the prices of forwards and futures are the same as long as the correlation between the underlying’s assets price and the risk-free rate is close to zero. In the case at hand, both the OTC contract and
the futures contract are for the same commodity (West Texas Intermediate or WTI sweet crude oil) and the same delivery date. Hence, the two contracts are excellent substitutes for one another, and there is no reason to believe that crude oil and interest rate levels are correlated. Consequently, the forward and futures prices should be the same: $98 per barrel.

(ii) If you buy 1,000 barrels of oil OTC, for delivery in a month, what will be your cash-flow today? In a month? Between today and the contract’s delivery date? Assume that this OTC forward contract is commodity-settled.

Answer:
As long as your OTC counterpart does not request any collateral, there is no cash-flow until delivery. At delivery, the long (i.e., you) pays $98,000 and receives 1,000 barrels of oil.

(iii) Would your answer change if the OTC contract were cash-settled (i.e., if it were an NDF or “non-deliverable forward”)?

Answer:
Yes.

In the NDF case, assuming once more that your OTC counterpart does not request any collateral, there would again be no cash-flow until the contract’s expiration. However, in contrast to the commodity-settled contract, the oil would not be delivered at maturity and the $98,000 payment for the oil would not be made either. Instead, the long (i.e., you) would receive or pay an amount of cash equal to the difference between the spot price of oil at maturity, $S_T$, and the initial forward price, $F_{0,T}$. The short would pay or get the opposite amount.

For example, if a month from now WTI sweet crude oil turns out to be trading spot at $100, then the long will receive from the short a cash settlement of ($100-$98) per barrel, or $2,000 for 1,000 barrels. Intuitively, the long gained because the price went up.

Alternatively, if in a month WTI crude is trading spot at $97 per barrel, then the long will pay $1,000 to the short. Intuitively, the long lost because the price dropped below $98.

(iv) a. Suppose that you are a small independent crude oil refiner and refine about ten thousand barrels a month. You have signed forward contracts committing you to sell your output at fixed prices. What price risk are you facing?

Answer:
Since you need to buy oil in a month, by definition you don’t have the oil. Hence, you will be hurt if the price of oil goes up, unless you hedge your purchase cost now.
b. To hedge that risk, you wish to hedge the cost of refining 10,000 barrels of oil that you will be receiving and paying for in a month. Should you go long or short? How would you carry out the hedge in practice?

**Answer:**

Since you will be hurt if the price of oil goes up, you have a short position in the underlying commodity (crude oil in this case). Hence, you need to take a long forward (or futures) position.

To do so, you **could** buy 10,000 barrels for one-month forward delivery; or, you **could** go long a one-month NDF on 10,000 barrels; or, you **could** take a long position in 10 NYMEX crude oil futures. In practice, most companies would go long the required number of WTI crude oil futures.

(iv) c Suppose once again that you are a crude oil refiner, and wish to hedge the cost of refining 10,000 barrels of oil that you will be receiving and paying for in a month. If you decide to use an OTC contract, does it matter whether you use an NDF or a commodity-settled contract to hedge?

**Answer:**

No. In both cases, assuming that your OTC counterpart does not request any collateral, there is no cash-flow until delivery. At delivery, here is what happens:

(a) With the commodity-settled contract, the long (i.e., you) must pay $980,000 and in exchange receives 10,000 barrels of oil. Done.

(b) With the NDF, you receive from your short counterpart $S_T - F_{0,T}$ per barrel (or you pay $S_T - F_{0,T}$ to the short if this difference negative). But then, you must still go out and buy the oil on the spot market. At what price? Well, the spot price, which is $S_T$. Summing up, your total cash-flow per barrel on day T is thus $S_T - F_{0,T} - S_T$, or $-F_{0,T}$. For example, if crude ends up trading at $102 per barrel in a month (day T), then you will get $(102 - 98)$ per barrel from the short in the NDF contract, or $40,000, and pay $1,020,000 to whomever you end up buying the crude from on the spot market. Your total cost is again $980,000 (= $1,020,000 – $40,000).

(iv) d. In the same situation, would you prefer using a commodity-settled futures instead of a commodity-settled forward (OTC) contract?

**Answer:**

A straightforward answer is “no.” As shown in the handout on marking to market, forward and futures transactions yield the same cash-flows in the end.

A slightly more elaborate answer is that “Cost-wise, no; but from a practical point of view, maybe.” Specifically, it depends on whether the refiner wants the oil delivered or not (and where); and, on whether you are worried about margin calls.
If you go to delivery with a futures position, then the only places where you can get the oil delivered are those locations that are specified as acceptable delivery points in the futures contract. These places may or may not be practical for you. With a forward contract, you should be able to negotiate a more convenient delivery point. Of course, nothing prevents you from closing out your futures position right before delivery, and taking spot delivery at the location of your choice – in which case, we again have a draw between the forward and the futures.

Another issue is what happens if you don’t have large enough cash piles to meet large margin calls. Suppose, for example, that crude oil prices drop a lot in the next week – say, to $78 per barrel. Then, if you went long, you will have to meet margin calls of $98-$78=$20 per barrel. That’s a $200,000 negative cash-flow. If you don’t have the cash handy, then you’ll be in trouble. Of course, in a month, it’s likely that the oil price would be low too, and therefore that your cash flow at futures delivery would be smaller than that at forward delivery. But timing is everything: the fact that you’d be OK in the end is irrelevant if you have run out of cash and gone broke in the meantime. That’s what happened to both LTCM and to MetallGesellschaft.

**Question 3. (10 points)**

(i) Consider the crude oil refiner of question 2(iv) but suppose that he has not sold his output forward. What hedging strategy would you recommend? (Hint: crack spreads, anyone?)

**Answer**

Oil refiners’ business is to refine crude oil by "cracking" it -- which produces, mostly, gasoline and heating oil/kerosene/diesel (plus some other by-products). In the situation being considered, the refiner is facing risk related to the difference between the prices at which he can sell his refined products (gasoline and diesel) and at which he purchases his main input (crude oil). Hence, oil refiners can lock in their processing margin (residual income) by entering into transactions called “crack spreads”.

Such spreads are created in commodity derivatives markets by going long crude oil futures and offsetting the position by going short gasoline and heating oil futures. The resulting spread position allows the investor to hedge against risk due to the offsetting nature of the securities.

**Note:** the risk related to the margin is quite real – to wit, during the summer of 2005, the effects of hurricanes in the Southeastern United States created large volatility in the crack spread.

(ii) Consider a soybean processor who has neither locked in the cost of his inputs (i.e., the actual grains) has not sold his output (i.e., soybean oil and soybean “meal”) forward. What hedging strategy would you recommend? (Hint: crush spreads, anyone?)

**Answer**

Soybean processors’ business is to “crush” it -- which produces, mostly, oil and “meal” (which is a flour made by grinding the solid residue of soybean oil production). In the situation being considered, the refiner is facing risk related to the difference between the prices at which he can sell his refined products (oil and meal) and at which he purchases his main input
(soybeans). Hence, a processor can lock in its processing margin (residual income) by entering into transactions called “crush spreads” – going long the soybean futures, and short both soybean oil and soybean meal.

**Question 4. (5 points)**

The direct spot quote for the Canadian $ in New York is C$1 = USD0.76; the 180-day swap rate is –2 points.

**a.** What accounts for the difference between the 2 rates? Explain.

**Answer**

Given the swap rate, the 6-month outright forward quote is C$1 = USD (0.76 – 0.02) = USD 0.74. From IRP, we know that a country’s currency (here, the U.S. $) will sell at a forward premium when interest rates in that country are lower than in the other country (here, Canada). In this question, you need more C$ to buy forward US$ than you do spot: thus, it must be that 6-month interest rates are higher in Canada than in the U.S.

**b.** In the absence of any other information, can you use the 180-day forward quote to forecast the direct spot quote for the Canadian $ in New York, 6 months from now? Explain briefly.

**Answer**

Based on the available info, the best you might say is: $0.74. This is because, under the assumption that markets are efficient and that there is no risk premium, the forward rate should be an unbiased predictor of the future spot rate.

This being said, to the extent that you are asked to make a prediction for 6-month hence, the forward is likely to be a bad forecasting tool. As discussed in class, uncovered IRP (i.e., using the forward to predict future spot rates) works much better at fairly long-term horizons (see also the paper by Meredith & Chinn on the Online Library) than at horizons of less than a year. A key reason is the existence of a time-varying (and hard to predict) risk premium embedded in the forward rate: \( F_{t,T} = \mathbb{E}[S_T] + \text{risk premium} \). Given this empirical reality, you might reasonable make an argument that the best 6-month forecast is not (unlike what many older finance textbooks might have suggested) the forward rate but, instead, the current spot rate of $0.76.

**Question 5. (10 points)**

Suppose that you are a trader of JP Morgan allowed to do arbitrage. Annualized six-month LIBOR for the Yen and the U.S. dollar are:

- Bid (deposit) – Ask (Borrow) ¥ at 0.5%-0.625%
- Bid (deposit) – Ask (Borrow) $ at 5.375%-5.5%

From a phone call to a trader at Daiwa Bank, you learn that Daiwa will let customers buy and sell ¥ spot at 100.00-50 ¥/1$. A trader at Barclays is simultaneously quoting bid and ask 6-month swap rates of –300 points (i.e., he will buy and sell ¥ 6-month forward at 97.00-50 ¥/1$).

Answer

There are two ways to carry out arbitrage (“arb”) strategies in this case:

1. either borrow $, convert them spot into Yen, deposit the Yen, and sell the Yen forward for $ in order to repay the dollar loan (in an attempt to make a small profit)

2. or borrow ¥, convert them spot into dollars, deposit the dollars, and sell the dollars forward for ¥ in order to repay the Yen loan (in an attempt to make a small profit)

The “brute-force” method to solve this problem is to try both ways, and see if either strategy generates a profit: that is how a computer-based arbitraging program would proceed. When proceeding manually, however, note that at most one (if any) strategy can yield a profit, the faster way is to try to assess whether strategy 1 or strategy 2 should be the profitable one.

It appears that you should be able to make small arbitrage (“arb”) gains, because covered IRP does not seem to hold in this case. To see this quickly, let us focus on the round numbers:

(i) the $ is selling at about a 3% 6-month forward discount to the Yen (the 3% figure is obtained by expressing the swap rate of 3 ¥/1$ (=97-100) as a fraction of the bid spot rate (100.00 ¥/1$);

(ii) the interest rate differential, however, is smaller: again concentrating on round figures, the IR diff is about 5% per year annualized (= 5.5%-0.5%), or 2.5% per six months.

Put differently, it looks as though the dollar is trading at too steep a forward discount to the Yen given the observed interest rate differential. This suggests the “direction” of the possible arbitrage: you need to buy low (buy dollars forward) and sell high (i.e., sell Yen forward). In other words, strategy 1 seems like the way to go.

Assuming that this is the right way to go, you know what else you need to do: in order to get the Yen that you’ll be delivering forward, you need to invest Yen for 6 months today; you get those Yen spot, by purchasing them with dollars. You don’t have dollars, so you borrow them. In sum, the arb “loop” is to borrow dollars at 5.5%, convert them spot for Yen at 100 (or 0.01 $/1¥), deposit the Yen at 0.5%, and sell the Yen forward at 97.50 (or 0.010256 $/1¥).

Formally, the forward rate implied by the interest rate differential and the spot rate is:

\[
F = S \frac{1 + i \frac{T}{360}}{1 + i' \frac{T}{360}} = 0.01$/1¥ \cdot \frac{1 + 0.055 \frac{T}{360}}{1 + 0.005 \frac{T}{360}} = 0.010249$/1¥
\]

Now compare this with the 6-month forward rate quoted directly by Daiwa: 0.010256 $ / 1¥. Clearly, you should sell Yen forward at 0.010256 $/1¥ and buy the Yen forward “synthetically” at 0.010249 $/1¥ by borrowing $ at 5.5%, buying ¥ spot with the borrowed dollars at 100¥/1$, and investing the ¥ at the rate of 0.5%.
To conclude, let us make sure that the cash-flows all work out:

<table>
<thead>
<tr>
<th>cash-flows today</th>
<th>cash-flows in 6 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. + 1$ (borrow 1$)</td>
<td>- 1.0275$ (loan repayment incl. 6-mo interest of 5.5%)</td>
</tr>
<tr>
<td>b. - 1$ (exchange $ spot for ¥)</td>
<td>none</td>
</tr>
<tr>
<td></td>
<td>none</td>
</tr>
<tr>
<td>c. - 100¥ (invest ¥ at 0.5% for 6 mo)</td>
<td>+ 100.25 ¥</td>
</tr>
<tr>
<td>d. none (sell ¥ forward for $)</td>
<td>- 100.25 ¥</td>
</tr>
<tr>
<td></td>
<td>+ 100.25 ¥</td>
</tr>
<tr>
<td>total 0</td>
<td>+ 0.0007 $</td>
</tr>
</tbody>
</table>

The cost is nothing (0 net cash-flow today). For every dollar borrowed, however, the sure gains in six months are 0.0007$, i.e., a 0.07% profit margin.

Note: As an added exercise, you should prove that the reverse strategy (borrowing ¥ at 0.625%, converting the ¥ spot for $ at 100.50 ¥/1$, investing the $ at 5.375% and selling forward the anticipated $ proceeds for ¥ at 97.00 ¥/1$) would lead to a loss.

b. Suppose you must borrow $1m on behalf of your employer, JP Morgan. What would be your total borrowing cost (i.e., what is the total number of $ you would pay on your $1m loan)? (Hint: what are your borrowing choices?)

**Answer**

Your borrowing choices are the following:

1. either **borrow $** at LIBOR (5.5%): the total $ cost in 6 months would be **$27,500**.

2. or create a similar pattern of cash-flows, borrowing in ¥, converting the ¥ into $, and locking in the $ cost of the ¥ loan through a forward contract. Here, the cost would be as follows:

   - you need $1m today, hence you borrow ¥100,500,000 and sell them spot for $1m (i.e., you buy $1m at the asked price of ¥100.50/1$)

   - in 6 months, you will need to pay back ¥100,814,063; you can lock in today the $ cost of this repayment by buying the Yen with (i.e., by selling) $1,039,320 six-month forward. The total $ cost would be: **$39,320**. The operation I have just described is called a swap.

   Since borrowing directly in $ is cheaper ($27,000 vs. $39,320), you should borrow $.
**Bonus Question.** (5 points – TBD if exam material)

Suppose the 3-month interest rate in Denmark is about 3.5%. Meanwhile, the equivalent interest rate in England is about 6.5%. All rates are annualized. What should be the annualized 3-month forward discount or premium at which the Danish krone will sell against the pound?

**Answer**

From covered interest rate parity, we know that the Danish krone (DKr) should sell at a premium against the pound approximately equal to the interest rate differential between the two countries, i.e., the krone should be trading at a premium of about 3% to offset the lower interest rate in Denmark.

Precisely, let $S_t$ and $F_{t,T}$ stand for the £ (GBP) spot and T-day forward prices of a Krone (DKK), respectively. Then, the percentage forward premium must be equal to:

$$
\frac{F_{t,T} - S_T}{S_T} = \frac{i_{GBP} \frac{T}{365} - i_{DKK} \frac{T}{360}}{1 + i_{DKK} \frac{T}{360}} = \frac{(6.5\% - 3.5\%) \frac{1}{4}}{1 + 3.5\% \frac{1}{4}} = 0.7435\%
$$

which is equivalent to 2.974% in annualized terms.

The formula can be rewritten to yield the percentage forward discount at which the £ should be trading against the Danish Krone:

$$
\frac{1}{F_{t,T}} - \frac{1}{S_T} = \frac{i_{DKK} \frac{T}{360} - i_{GBP} \frac{T}{365}}{1 + i_{GBP} \frac{T}{365}} = \frac{(3.5\% - 6.5\%) \frac{1}{4}}{1 + 6.5\% \frac{1}{4}} = -0.738\%
$$

i.e., the £ should trade at a 0.738% 3-month forward discount against the DKr. **On an annualized basis, we should have the £ trading at a 2.952% 3-month forward discount.**

**Note:** because we are not given the exact number of days in the period considered, I have chosen to approximate the 3-month period by $\frac{1}{4}$ of a year (i.e., 90/360 days for the Krone or 91/365 days for the £).