The Theory of Bank Risk-Taking and Competition Revisited

John H. Boyd and Gianni De Nicoló*

ABSTRACT

There is a large literature that concludes that—when confronted with increased competition—banks rationally choose more risky portfolios. We argue that this literature has had a significant influence on regulators and central bankers. We review the empirical literature and conclude that the evidence is best described as “mixed.” We then show that existing theoretical analyses of this topic are fragile, since there exist fundamental risk-incentive mechanisms that operate in exactly the opposite direction, causing banks to become more risky as their markets become more concentrated. These mechanisms should be essential ingredients of models of bank competition.

* John H. Boyd is Kappel Chair in Business and Government, Finance Department, University of Minnesota. Gianni De Nicoló is Senior Economist in the Monetary and Financial Systems Department of the International Monetary Fund. The views expressed in this paper are those of the authors and do not necessarily represent the views of the International Monetary Fund or of the Federal Reserve System.
It is a hoary notion in banking that “excessive competition” can lead to socially undesirable outcomes in the form of bank failures, runs, and panics. As recently noted by a prominent central banker, “The legislative reforms adopted in most countries as a response to the banking and financial crises of the 1930s shared one basic idea which was that, in order to preserve the stability of the banking and financial industry, competition had to be restrained. This fundamental proposition was at the root of the reforms introduced at that time in the United States, Italy, and most other countries” Padoa-Schioppa (2001, p. 14). The basic idea is that, when banks can earn monopoly rents, they become relatively conservative. Their banking charter is valuable and, thus, they shun the risk of bankruptcy, because bankruptcy would cause the loss of a valuable charter. At least one widely-cited empirical study provides support for this view, as do a host of theoretical analyses to be reviewed momentarily. Yet, the weight of empirical evidence regarding indicators of banking market structure and bank risk-taking is mixed, with no clear consensus.

This paper shows that there exists a fundamental risk-incentive mechanism that operates in exactly the opposite direction, causing banks to become more risky as their markets become more concentrated. This mechanism exists on the asset side of the balance sheet and has been unmodeled in widely-cited studies that focus on deposit market competition. Ceteris paribus, as competition declines banks earn more rents in their loan markets by charging higher loan rates. In themselves, higher loan rates would imply (weakly) higher bankruptcy risk for bank borrowers. This effect is further reinforced by moral hazard on the part of borrowers who, when confronted with higher interest costs, optimally increase their own risk of failure.
The remainder of the paper is organized as follows. Section I reviews the theoretical literature and illustrates its influence on regulators and central bankers. Section II reviews the empirical literature. Section III presents our base model of deposit competition. Section IV extends the model to competition in both loan and deposit markets. Section V outlines some other fragilities of the existing paradigm. Section VI concludes by outlining proposed model extensions capable of embedding the identified risk-incentive mechanisms in more general models of bank competition.

I. Literature Review

Modern models of bank risk-taking feature the role of deposit insurance and other government interventions that result in moral hazard, which distorts banks’ risk incentives. Broadly speaking, this literature concludes that deposit insurance results in an incentive to intentionally take on risk of failure, possibly without limit. The incentive is due to a payoff structure in which large gains go to bank shareholders and large losses to the government. Conceptually, the way to solve this problem is to provide bank shareholders with a stake in the firm that is sufficiently high, so that their incentives are aligned with those of the deposit insurer.¹

There are two related but different ways to implement such a policy: the policy-maker can either force bank shareholders to hold large stakes against their will; or it can give them large stakes that they will hold voluntarily. Theoretically, for such policies to work, either the insured bank’s ability to increase risk of failure must be bounded, or higher risk of failure must be “costly” in the sense that it results in decreasing expected asset returns (although
Kareken and Wallace (1978) show cases in which such policies will not work). The forcing policy refers to mandatory capital requirements in which the regulator imposes constraints on the use of financial leverage. Such policies are the essence of the BIS capital standards and are almost universally employed by bank regulators. However, there is a continuing debate over whether such policies are efficient, or even effective (see Hellman, Murdoch and Stiglitz (2000) and Marshall and Prescott (2001)).

The second regulatory approach is to give bank equity holders a sufficiently large stake in their bank. This is accomplished by a policy of intentionally allowing banks to earn monopoly rents, so that the franchise becomes valuable and going broke costly. Such policies have been analyzed in Allen and Gale (2000); Hellman, Murdoch and Stiglitz (2000); Keeley (1990); and Repullo (2003). In each of these studies, as banks earn more rents in deposit markets, their equilibrium risk of failure declines. Allen and Gale (2000) study an environment in which banks choose a parameter that determines the default risk of their assets, and are Cournot-Nash competitors in deposit markets. They show that as the number of deposit market competitors becomes arbitrarily large, the optimal risk of failure reaches a maximum. Hellman, Murdoch, and Stiglitz (2000) study a policy of deposit interest rate ceilings and find that such regulation decreases risk of failure. In their model, this policy is more effective than capital regulation and can, under certain conditions, be an optimal policy.²

These studies share one important assumption. It is that banks’ optimal asset allocations are determined by solving a *portfolio problem* that takes asset prices and return distributions as given. Although many other banking studies make similar assumptions, there
is a large and growing literature that does not. A common alternative assumption is that banks solve an *optimal contracting problem* in which the actions of borrowers are unobservable, or observable at cost.³ We will show that in the latter kind of moral hazard model of a bank, competition plays a new and different role; further, we present a simple moral hazard model in which, as bank markets become more competitive, risk of failure unambiguously declines⁴.

What explains this radical reversal of the predictions of theory? Intuitively, in the portfolio model, as deposit markets become more concentrated, banks use their market power to become more profitable. Resultantly, they become less eager to seek low probability, high return outcomes. Any direct effect of loan market competition is ignored. In the contracting model, on the other hand, banks compete in both deposit and loan markets. Less competition means more rents earned in deposit markets (as before), but also means more rents earned in loan markets. Obviously, higher loan rates are charged to bank customers as concentration increases. In themselves, higher loan rates imply (weakly) higher bankruptcy rates for borrowers, but the loan market risk channel is further enhanced by moral hazard on the part of borrowers. When confronted with higher loan rates charged by banks, they optimally adjust their investment policies in favor of more risk.

Notice that it is exactly the same kind of mechanism that is influencing both banks and their loan customers, but having opposite effects. Less competition in banking results in higher deposit rates, bank profits go up, and banks intentionally seek less risk. At the same time, less competition in banking means higher loan rates, borrower profits go down, and they intentionally seek more risk. In the simple moral hazard model to be presented, the loan
market effect dominates and increasing competition unambiguously results in lower bank risk. We are not claiming that this is a general result. As we will see momentarily, empirical tests of the link between competition and risk-shifting have produced mixed conclusions. All we are claiming is that the loan market channel exists and \textit{a priori} is just as important as the deposit market channel.

In a moment, we turn to our analysis. Before doing that, we briefly argue that the recent theoretical literature has had some significant effect on policy-makers. Or put another way, we argue that this topic is of more than theoretical interest.

\textbf{A. Policy Ramifications}

We believe that the body of research just reviewed has had a material impact on bank regulators and central bankers. Specifically, we believe there is a widely-held view among policy makers that reduced competition in banking is not necessarily bad because, other effects notwithstanding, reduced competition results in a more stable banking industry. Obviously, policy makers are aware of other social costs that may be associated with bank rent-earning. That is not the point. In the environment of the last several decades, with banking crises occurring around the globe, policy-makers might be expected to pursue risk-abatement strategies even if there were some “attendant costs.” Carletti and Hartmann (2003) argue this same point. “Finally, it may be that the very influential ‘charter value hypothesis’ (see e.g. the discussion of Keeley [1990] and others below), saying roughly that a too competitive banking sector will be prone to instability, has convinced some countries to
counterbalance the competition-oriented antitrust review with a stability-oriented supervisory review of bank mergers.”

For obvious reasons, policy spokespersons are loath to publicly state that they encourage monopoly rent-earning by banks so as to stabilize that sector. However, there is a historical track record of events that is strongly suggestive. In the United States, the Federal Reserve System has been consistently permissive of bank mergers even when they involved the very largest banks. Yet, there is a large literature indicating that mergers of very large banks produce little or nothing in the way of scale economies, and may even produce diseconomies. (e.g. Group of Ten (2001), De Nicoló (2000), and the literature cited therein). There is also much suggestive evidence based on the treatment of banks in the many banking crises around the world. Local and international agencies have pursued aggressive merger policies in almost all crisis situations, even in bank markets that were already highly-concentrated by any standard. In evaluating banking crises around the world in the last twenty years, Caprio and Klingebiel (2000) conclude that “achieving better banking will require the use of both carrot— in the form of profitable opportunities for banking (our italics)—and stick—such as prompt replacement of bad managers, substantial losses for (and replacement of ) owners and more mobile deposits. Such mechanisms will ensure that bankers take risks, but only risks that are prudent” (p. 296).

II. Empirical Relevance

There exists a large literature examining the relationship between competition in banking markets and various proxy measures of bank risk exposure. As discussed below this
literature has produced mixed findings. A drawback of much of this work is that it employs risk proxy measures that are only indirectly related to the probability of bank failures.

An early literature on U.S. banking markets (reviewed by Rhoades and Rutz (1982)) tests John Hicks’ conjecture that “the best of monopoly profits is a quite life.” For samples of U.S. (unit) banks in the 1960s and 1970s, a negative relationship is found between bank concentration and proxy indicators of risk-taking including: profit volatility, debt / asset ratios, and non-performing loan ratios. Later, in an influential study, Keeley (1990) presents evidence that the increased competition prompted by relaxation of state branching restrictions in the 1980s induced U.S. large bank holding companies to increase their risk profiles, as proxied by estimates of market capital-to-asset ratios and actual interest costs on large CDs. Yet, using a substantially larger sample, Jayaratne and Strahan (1998) find that deregulation was followed by sharp reductions in loan losses, contrasting Keeley’s earlier result. Even more recently, Dick (2003) provides some evidence of increased charged-off losses and loan loss provisions following deregulation in the 1990s, again apparently at odd with Jayaratne and Strahan’s results. In sum, this strand of work uses indirect proxy measures of the risk of failure and produces mixed and sometimes conflicting results.

Yet another strand of the literature has employed defensible measures of failure probabilities, but weak measures of competition. Boyd and Runkle (1993) and De Nicoló (2000) relate indicators of bank failure probabilities to bank size, not to market-wide competition measures, such as Herfindahl Indices or concentration ratios. Arguably, the size variable is likely to be correlated with market power, so the findings of these studies are at least suggestive. Boyd and Runkle (1993) find that failure probabilities are essentially unrelated to bank size. De Nicoló (2000) reports a positive and significant relationship
between bank size and failure probabilities for the United States, Japan, and several European countries.

Two recent studies examine the relationship between competition in bank markets (as measured by a concentration ratio) and bank fragility. Both studies assume that a nation defines a banking market, and both look at the cross-country relationship between concentration and proxy measures of banking fragility. In other respects these studies are methodologically quite different, and they reach somewhat different conclusions.

Beck, Demirguc-Kunt, and Levine (2003) define banking fragility as the occurrence of a banking crisis, where the definition and dating of a banking crisis is based on Caprio and Klingebiel (1999). Their data set is a panel of seventy countries over the period 1980 to 1997, and they estimate a Logit probability model in which the dependent variable is the probability of a banking crisis. The bank concentration ratio enters as an explanatory variable, along with a host of macroeconomic and structural controls. They find that the concentration ratio is negatively and significantly associated with the probability of a banking crisis. They provide a number of different specifications and robustness checks, and this finding seems quite solid. However, the authors are extremely cautious in interpreting it, primarily because they find that other measures of competition have the opposite effect on banking crisis probability. Most importantly, they find that restrictions on new bank entry significantly increase the probability of a crisis. This leads the authors to question if concentration ratios (which have long been a standard measure of market power in this literature) can be used as simple proxy measures for market power.

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De Nicoló et al. (2003) take a different approach to empirically representing banking system fragility. Instead of trying to date banking crisis beginnings and endings, they construct a probability of failure measure for the five largest banks in a country, viewed as an indicator of systemic risk. Their measure of competition is a five-bank concentration ratio, and this enters alongside control variables for inflation, region of the world, real GDP growth, and a government intervention variable that is intended to capture the effect of government bailouts on banks’ profit distributions. In several different specifications, they find that the probability of failure measure is positively and significantly associated with bank concentration, meaning that ceteris paribus, a more concentrated banking industry is more prone to banking fragility.

It is beyond the scope of this study to more fully review this literature or to speculate as to the reason for the many differences in findings. The point is that there is no clear consensus in the empirical literature on this topic, and thus no strong support for the predictions of the existing theoretical literature. We next turn to our model.

### III. A Model of Bank Competition

We begin with a model presented by Allen and Gale (chapter 8, 2000), which is used for our “base case.” Then, we will modify the model to allow for the existence of a loan market.

The economy lasts two dates: 0 and 1. There are two classes of agents, banks and depositors, and all agents are risk-neutral.

**Banks**
There are $N$ banks that have no initial resources but have access to a set of constant return-to-scale risky technologies indexed by $S$. Given an input level $y$, the risky technology yields $Sy$ with probability $p(S)$ and 0 otherwise.

**Assumption 1.** $p(S)$ satisfies: $p(0) = 1, p\left(\bar{S}\right) = 0$, $p' < 0$ and $p'' \leq 0$ for all $S \in [0, \bar{S}]$.

Thus, $p(S)S$ is a strictly concave function of $S$ and reaches a maximum $S^*$ when $p'(S^*)S^* + p(S^*) = 0$. Given an input level $y$, increasing $S$ from the left of $S^*$ entails increases in both the probability of failure and expected output. To the right of $S^*$, the higher $S$, the higher is the probability of failure and the lower is expected output. The bank's (date 0) choice of $S$ is unobservable to outsiders. At date 1, outsiders can only observe and verify at no cost whether the investment’s outcome has been successful (positive output) or unsuccessful (zero output). By assumption, contracts are simple debt contracts. Observe that the bank has complete control over the choice of risk-taking.

**Depositors**

The total supply of deposits is represented by an upward sloping inverse supply curve, denoted by $r_D(\cdot)$, with,

**Assumption 2.** $r_D(\cdot)$ satisfies: $r_D(0) \geq 0, r_D' > 0, r_D'' \geq 0$.

Total deposits of bank $i$ are denoted by $D_i$, and total deposits by $\sum_{i=1}^{N} D_i$. 
Deposits are insured, so that the relevant supply does not depend on risk, and, for this insurance, banks pay a flat rate deposit insurance premium, denoted by \( \alpha > 0 \).

Banks compete for deposits in a Nash fashion. We assume that the rate of interest on deposits is a function of total deposits\(^6\): 

\[
r_D = r_D \left( \sum_{i=1}^{N} D_i \right).
\]

In a Nash equilibrium, each bank \( i \) chooses \((S_i, D_i) \in \left[ 0, \bar{S} \right] \times R\), that is the best response to the strategies of other banks. The pair \((S_i, D_i)\) is chosen to maximize

\[
p_i(S_i) \left( S_i - r_D \left( \sum_{i=1}^{N} D_i \right) - \alpha D_i \right). \quad (1)
\]

Necessary conditions for an interior equilibrium \((D_i > 0 \text{ and } S \in (0, \bar{S}))\) are

\[
p'(S_i) \left( S_i - r_D \left( \sum_{i=1}^{N} D_i \right) - \alpha \right) D_i + p(S_i) D_i = 0, \quad (2)
\]

\[
p(S_i) \left( S_i - r_D \left( \sum_{i=1}^{N} D_i \right) - r_D' \left( \sum_{i=1}^{N} D_i \right) - \alpha \right) = 0. \quad (3)
\]

In a symmetric interior equilibrium \((S_i, D_i) = (S, D) > 0\) for all \( i \) and \( p(S) > 0 \). Hence, the above conditions reduce to:
\[ p'(S)(S - r_D(ND) - \alpha) + p(S) = 0, \quad (4) \]

\[ S - r_D(ND) - r'_D(ND)D - \alpha = 0. \quad (5) \]

Allen and Gale prove that the above system has a unique solution. In the Appendix, we prove the following

**PROPOSITION 1:** In a symmetric interior equilibrium, the equilibrium level of risk shifting \( S \) is strictly increasing in \( N \). As \( N \to \infty \), \( S \to \bar{S} \).

As discussed in Section V, the simple monotonic relationship between number of banks in a market and equilibrium risk-seeking in this base case model is not robust to different, but plausible, alternative assumptions. Before turning to that, we next show that the competition- risk-seeking relationship is reversed by the introduction of a loan market.

**IV. The Model with a Loan Market**

In the above setup, banks allocate their assets by solving a portfolio problem that trades off risk of failure and expected returns. For many purposes, this stylization is perfectly acceptable. But it ignores the existence of a loan market. For present purposes such stylization means that changes in bank market structure (\( N \)) can only affect asset allocations indirectly; e.g., through their effect on deposit costs. This is unrealistic and important for, in effect, it amounts to permitting the number of competitors in the deposit market to change, while holding the number of competitors in the loan market fixed. The Hellmann *et. al.* study has the same structure and is susceptible to the same criticism. As we show next, it is
relatively easy to allow for the same $N$ banks competing for both deposits and loans so that, as $N$ changes, both markets are symmetrically affected.

Consider many entrepreneurs, who have access to projects of fixed size, normalized to 1, with the two-point random return structure previously described. They borrow from banks, who cannot observe their risk shifting choice $S$, but take into account the best response of entrepreneurs to their choice of the loan rate. In this case, as opposed to the previous case, the bank has no direct control over the riskiness of borrower’s projects. Given a loan rate $r_L$, entrepreneurs choose $S \in [0, \bar{S}]$ to maximize:

$$p(S)(S - r_L).$$

(6)

By the strict concavity of the objective function, an interior solution to the above problem is characterized by

$$h(S) = S + \frac{p(S)}{p'(S)} = r_L.$$  

(7)

Note that by (7), an increase in the interest rate on loans causes an entrepreneur to choose more risk through an increase in $S$.

Let $L$ denote the total amount of loans. The inverse demand for loans satisfies

**Assumption 3.** $r_L(0) > 0, r'_L < 0, r''_L \leq 0$ and $r_L(0) > r_D(0),$

with the last condition ensuring the existence of equilibrium. This inverse demand for loans can be generated by a population of potential borrowers whose reservation utility to operate the productive technology differs. Consistent with our treatment of deposit market
competition, we assume that the rate of interest on loans is a function of total loans:

\[ r_L = r_L(L). \]

Banks have no equity in this model, so that the balance sheet identity requires that

\[ L = \sum_{i=1}^{N} D_i. \]

In a Nash equilibrium, each bank chooses deposits to maximize profits, given similar choices of the other banks and taking into account the entrepreneurs’ choice of \( S \).

Note that the assumption that banks lend to entrepreneurs whose returns are perfectly correlated is equivalent to assuming that the risk associated with each loan can be decomposed into a systemic and idiosyncratic component, and that with a large number of entrepreneurs, the idiosyncratic component can be perfectly diversified away. This assumption is similar to the assumption of a bank portfolio composed of perfectly correlated risks used in section III, and, as detailed in what follows, is sufficient to generate, *ceteris paribus*, a direct relationship between loan interest rates and banks’ probability of failure.

Thus, bank \( i \) chooses \( D_i \) to maximize

\[
p(S) \left( r_L \left( \sum_{i=1}^{N} D_i \right) - r_D \left( \sum_{i=1}^{N} D_i \right) - \alpha D_i \right), \tag{8}
\]

subject to

\[
h(S) = S + \frac{p(S)}{p'(S)} = r_L \left( \sum_{i=1}^{N} D_i \right), \tag{9}
\]
where (9) reflects the equality of total loans with total deposits and the fact that borrowers will choose an optimal (for them) value of $S$. Let $S\left(\sum_{i=1}^{N} D_i\right)$ denote the function implicitly defined by (9).

Thus, bank $i$ chooses $D_i$ to maximize

$$\Pi(D_i) = p\left(S\left(\sum_{i=1}^{N} D_i\right)\right)\left[r_L\left(\sum_{i=1}^{N} D_i\right)D_i - r_D\left(\sum_{i=1}^{N} D_i\right)D_i - \alpha D_i\right]$$

subject to

$$0 \leq S\left(\sum_{i=1}^{N} D_i\right) \leq \bar{S}. \quad (11)$$

Let $Z$ denote total deposits. In the Appendix, we prove the following

**PROPOSITION 2:** In a symmetric interior Nash equilibrium, the equilibrium level of risk shifting $S$ is strictly decreasing in $N$. As $N \to \infty$, the Nash equilibrium converges to the competitive outcome, $r_L(Z) - r_D(Z) - \alpha = 0$.

The intuition behind Proposition 2 is straightforward. As banks obtain increased market power in the loan market, they use it to raise loan rates *ceteris paribus*. Entrepreneurs optimally respond (for reasons that are well-known), by increasing the risk of their investment projects. The banks are aware that this response will occur and take it into account in their choice of a loan rate. Exactly this kind of interaction has been featured in most modern models of bank lending that feature moral hazard on the part of entrepreneurs.
However, this important interaction is absent in all models that treat bank lending as a portfolio problem so that changes in \( N \) do not directly affect asset allocations.

V. More on the Fragility of Existing Theory

As shown in Boyd and De Nicoló (2003), the simple monotonic relationship between the number of banks and equilibrium risk-seeking (Proposition 1) breaks down if fixed bankruptcy costs are introduced, or if deposit insurance is fairly priced so that a deposit insurer breaks even.

In the case of fixed bankruptcy costs, the key reason behind the reversal of the risk-shifting effect is the dependence of the risk-shifting choices on bank size. Such dependence arises from fixed bankruptcy costs whenever the average size of banks is decreasing in \( N \). As competition increases, banks’ bankruptcy costs are rising at the margin, inducing banks to take on less risk, \( ceteris paribus \). Thus, the benefits of risk shifting will eventually be offset by bankruptcy costs, once the number of banks in a market passes some critical threshold. Thus, under these assumptions, the relationship between risk-shifting behavior and competition is not monotone.

If bankruptcy costs are introduced in the model with a loan market, its implications for risk unavoidably become much more complicated. That is because bankruptcy costs need to be introduced for both banks and entrepreneurs, and the effects of the two are quite different. As discussed in the conclusion, this is on our agenda for future research.

In the case of fairly priced deposit insurance, the simple monotonic relationship between \( N \) and \( S \) again disappears. As we show in Boyd and De Nicoló \textit{op. cit.}, there may be one equilibrium, several, or no equilibria, depending on \( N \). Thus, with this change in
model assumptions, the relationship between risk-shifting behavior and competition can depend on the number of banks in a rather complicated way.

VI. Conclusion

The existing theoretical literature concludes that, when confronted with increasing competition, moral hazard is exacerbated and banks intentionally take on more risk. We have reviewed this literature and shown that a positive relationship between the number of bank competitors and risk-seeking is fragile. In particular, it makes an enormous difference when one allows for the existence of loan markets and requires that there be the same number of banks competing for both deposits and for loans. In this version of the model, we assume that borrowers entirely determine project risk, conditional on the loan rate set by banks. In effect, we take the bank portfolio problem and transform it into a contracting problem with moral hazard. With that modeling structure, banks use increasing market power to raise loan rates and, when confronted with increased funding costs, borrowers optimally choose higher risk projects.

In our models, banks strategic interaction has been modeled a la Cournot for the sake of transparency and simplicity. Other forms of bank strategic interaction, such as price competition in the context of locational models, have been used in the banking literature. It might be useful to see how our results are affected by other forms of market interaction.

It seems clear that a continuous asset return distribution is more general than the degenerate asset return distribution employed here. Moreover, it makes a difference when the researcher allows for bankruptcy costs in some bad states of the world. Thus, this is an obvious extension of our work. In addition, future modeling efforts should include elements
of both what we have called a “portfolio model” and a “contracting model.” In reality, banks hold large portfolios of debt and equity securities traded in markets in which they are price-takers. At the same time, banks hold many different kinds of loans (with different potential for moral hazard problems). Both kinds of activities need to be included in the same model, and that is something we have not done. The modeler could also include a more complicated contracting environment between banks and borrowers (or between banks and the deposit insurance agency), for example, by including technologies for costly monitoring. It seems to us that these kinds of complications are likely to be of second-order importance in the context of optimal contracting, for they will not change the nature of risk-seeking incentives, either for banks or for bank borrowers.

In addition, it would be a good idea to allow for the issuance of bank equity claims. We have not considered this extension here, but it is potentially important for two reasons. First, equity claims are not insured and thus (unlike insured deposits) their expected returns depend on default probabilities. Second, equity claims are traded in “the equity market,” and, to a first approximation, the number of competitors in this market is independent of the number of banks.

General equilibrium considerations are likely to be particularly important. Allen and Gale (2004) show that in a general equilibrium model with financial intermediation, asymmetric information, and incomplete contracts, the perfectly competitive allocation is constrained efficient and financial instability is necessary for efficiency. Thus, there is no tradeoff between competition and instability. Boyd, De Nicoló and Smith (2004) show that in a monetary general equilibrium economy, the probability of a banking crisis may be either higher under competition or under monopoly, depending on the rate of inflation. Thus, the
relative crisis probabilities cannot be determined independently of the conduct of monetary policy.

Our research plans include studying these extensions in a general equilibrium environment. Until that work is done by us or others, we are unaware of any compelling theoretical arguments that banking stability decreases (or increases) with the degree of competition in bank markets.
APPENDIX

PROPOSITION 1: In a symmetric interior equilibrium, the equilibrium level of risk shifting $S$ is strictly increasing in $N$. As $N \to \infty$, $S \to \bar{S}$.

Proof of Proposition 1: The comparative statics of this model can be readily analyzed by taking the total differential of the system (4)-(5) with respect to the risk shifting parameter $S$ and the amount of total deposits, denoted by $Z \equiv ND$. Let $h(S) \equiv S + \frac{p(S)}{p'(S)}$. System (4)-(5) can be re-written as:

\begin{align*}
  h(S) - r_D(Z) - \alpha &= 0, \\  S - r_D(Z) - r'_D(Z)Z / N - \alpha &= 0.
\end{align*}

Taking the total differential with respect to $S$ and $Z$, we get

\begin{align*}
  h'(S) dS - r'_D(Z) dZ &= 0, \\  dS - \left( r'_D(Z)(1+1/N) + r''_D(Z)Z / N \right) dZ &= -r'_D(Z)Z / N^2 dN.
\end{align*}

By the maintained assumptions, and since $h'(S) = 2 - \frac{p(S)p''(S)}{(p'(S))^2} \geq 2$, the determinant of the above system, denoted by $\Delta$, satisfies

\begin{equation}
  \Delta \equiv r'_D(Z)(1 - h'(S)) - \frac{h'(S)}{N}\left( r'_D(Z) + r''_D(Z)Z \right) < 0.
\end{equation}
By Cramer rule,

\[
\frac{dS}{dN} = -\frac{1}{\Delta} r'_D(Z)^2 Z > 0 \quad \text{and} \quad \frac{dZ}{dN} = -\frac{1}{\Delta} h'(S) r'_D(Z) Z > 0. \quad (A6)
\]

Thus, total deposits increase and the risk shifting parameter \(S\) increases with \(N\). The latter is the standard risk shifting effect described in much of the literature. Allen and Gale also prove that as \(N \to \infty\), \(S \to \hat{S}\). Q.E.D.

These statements can be illustrated by the following parametric example. Let \(p(S) = 1 - AS\) defined on \([0, A^{-1}]\), and \(r_D(X) = B_0 + B_1 X\), with both coefficients strictly positive, and assume \(1/A > B_0 + \alpha\). Then, solving conditions ((A.1) and (A.2) gives

\[
S = \frac{1}{A} - \frac{1}{A} \frac{A - B_0 - \alpha}{N + 2} \quad \text{and} \quad Z = \frac{1}{A} \frac{A - B_0 - \alpha}{B_1} \frac{N}{N + 2}. \quad \text{Clearly, the risk of failure increases, and total deposits increase, as the number of banks increases.}
\]

**PROPOSITION 2:** In a symmetric interior Nash equilibrium, the equilibrium level of risk shifting \(S\) is strictly decreasing in \(N\). As \(N \to \infty\), the Nash equilibrium converges to the competitive outcome, \(r_L(Z) - r_D(Z) - \alpha = 0\).

**Proof of Proposition 2:** Using \(Z = ND\), necessary and sufficient conditions for a symmetric interior equilibrium are

\[
h(S) = r_L(Z), \quad (A7)
\]
\[ r_L(Z) - r_D(Z) - \alpha - F(Z, N) = 0, \quad (A8) \]
\[ r'_L(Z) - r'_D(Z) - F_1(Z, N) < 0, \quad (A9) \]

where

\[ F(Z, N) = \frac{(r'_D(Z) - r'_L(Z)) p(S(Z)) Z}{p'(S(Z)) S'(Z) Z + p(S(Z)) N}. \quad (A10) \]

The term \( F(Z, N) \) incorporates “monopoly rents.” By differentiating equation (A7) with respect to \( Z \), we obtain \( S'(Z) = (h'(S))^{-1} r'_L(Z) < 0 \). Thus, \( F(Z, N) > 0 \) and \( F_2(Z, N) < 0 \).

Totally differentiating equation (A8), we get

\[ \left( r'_L(Z) - r'_D(Z) - F_1(Z, N) \right) dZ - F'_2(Z, N) dN = 0. \quad (A11) \]

Thus, for any finite \( N \)

\[ \frac{dZ}{dN} = \frac{F'_2(Z, N)}{r'_L(Z) - r'_D(Z) - F_1(Z, N)} > 0; \quad (A12) \]

i.e., total deposits increase as the number of banks increases.

Totally differentiating equation (A7),

\[ \frac{dS}{dN} = \frac{r'_L(Z) dZ}{h'(S) dN} < 0, \quad (A13) \]

since \( h'(S) > 0 \). Moreover, \( F(Z, N) \to 0 \) as \( N \to \infty \). Q.E.D.
The following example illustrates Proposition 2 for an economy that satisfies all maintained assumptions. Let \( p(S) = 1 - \beta S \), \( r_L(Z) = \frac{1}{A} - \beta Z \) and \( r_D(Z) = \gamma Z \), with all parameters strictly positive. Using (A7), entrepreneurs’ optimal risk shifting schedule is given by \( S(Z) = \frac{1}{A} - \frac{\beta}{2} Z \) (of course, \( S'(Z) = -\frac{\beta}{2} \)), and \( p(S(Z)) = \frac{A\beta}{2} Z \). It is straightforward to verify that, for this economy, equation (A9) is always satisfied for any choice of bank deposits satisfying equation (A8). Under these assumptions, equation (A8) yields \( Z = \frac{-\beta_0}{\beta_1 + \gamma} \frac{N + 1}{N + 2} \), with \( p(S(Z)) \) always between 0 and 1 for any value of \( N \). Thus, total deposits increase, and the risk-shifting variable \( S \) decreases, as the number of banks increases. If the loan rate intercept is different from \( 1/A \), equation (A8) yields a quadratic equation in \( Z \). It is straightforward to verify that there is only one (strictly positive) solution, and this solution increases as the number of banks increases. Thus, Proposition 2 can be verified for all parameter configurations that yield a symmetric interior Nash equilibrium.
REFERENCES


Allen, Franklin, and Douglas Gale, 2004, Financial intermediaries and markets,

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Endnotes

1 Another approach is to “correctly” price deposit insurance so that increasing risk of failure results in increasing insurance costs (Merton (1977)). However, this is hard to implement in practice given the opaqueness of banks, and is unlikely to be optimal policy anyway, if the general equilibrium effects of regulatory policy are taken into account (Boyd, Chang and Smith (2002)).

2 There is another very different literature that has identified a possible cost of competition in banking. This literature considers the effect of market power on banks' willingness to lend to small or new firms (for example, Petersen and Rajan (1995), or Berger and Udel (1995)). However, this work is not directly related to the issue of risk incentives that we address here.

3 So many papers fit this description it is impossible to cite them here. See Gorton and Winton (2002) for an excellent review of this literature.

4 In the terminology of agency theory, while a bank is an “agent” in relation to its depositors, it is a “principal” in relation to its borrowers. The literature reviewed focuses on the bank-depositor relationship but ignores the bank-borrower relationship. When the bank is treated as an agent, more competition decreases the agent’s (bank’s) payoff and increases the social loss, as measured by the distance between first best and second best expected output levels. When the borrower is treated as an agent and the bank is both a principal (in relation to the borrowers) and an agent (in relation to the depositors) but, as we assume, does not directly choose the level of risk-taking, then more competition in both the loan and deposit market increases agent’s (borrower’s) payoff and decreases the social loss. We thank an anonymous referee for having pointed out to us this insightful analogy.

5 The relationship between bank concentration ratios and the Panzar-Ross measure of competitiveness is also empirically unsettled. For large samples of banks in many industrialized countries, Bikker and Haaf (2002) and Corvoiser and Gropp (2002) find that some concentration measures are valid proxies of competitive conditions, while Claessens and Laeven (2004) find no evidence that lower concentration may be indicative of higher competition.
If the market is replicated by assuming that the rate of interest on deposits is a function of deposits per bank, as in Allen and Gale, an increase in competition is represented by increasing the number of banks and depositors at the same rate. For present purposes, it makes no difference at all in terms of risk incentives.