Reverse Survivorship Bias*

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Preliminary, comments welcome

ABSTRACT

Mutual funds often disappear following poor performance. When this poor performance arises from negative idiosyncratic shocks, the alpha estimate of a disappeared fund is too low. This performance-survival correlation biases the observed alpha distribution downwards relative to the true distribution. I develop and estimate a structural model in which investors learn about mutual fund alphas to measure this effect. I find that the bias in the mean of the observed alpha distribution is approximately 1% per year. Although the majority of fund managers still have negative alphas, the average alpha is not nearly as low as what the fund-level estimates suggest. This reverse survivorship bias affects all studies that use fund-level regressions to draw inferences about fund managers’ abilities.

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The typical survivorship-bias argument starts from the observation that mutual funds often disappear following poor performance. Thus, a study that conditions on fund survival overstates mutual fund performance. In this paper I show that the correlation between poor performance and fund disappearance induces another pattern with the opposite sign: mutual fund alphas estimated from a survivorship-bias free data set are biased downwards relative to the true distribution of alphas.

The mechanics of this bias are transparent in a setting in which investors learn about fund alphas. (As I discuss below, the bias arises even in absence of learning. The only requirement is that funds tend to disappear following poor performance.) Suppose, for the sake of an argument, first, that investors have a prior belief that a fund’s alpha is zero; second, that the true alpha is fixed; and third, that investors abandon a mutual fund (and the fund shuts down) when their posterior belief is that the fund alpha is less than $-\bar{\alpha}$. Each month investors update their beliefs about the alpha based on the risk-adjusted return. If this return is positive, investors infer skill. If the return is negative, investors infer less skill. The posterior mean always lies between the prior mean and the realized return. As a consequence, if the posterior mean ever falls below $-\bar{\alpha}$ and the fund disappears, then the observed alpha must have been strictly lower than $-\bar{\alpha}$. If not, the posterior mean, which is an unbiased estimate of the fund’s true alpha, could not have crossed this threshold. The resultant positive gap between the true alpha and the observed alpha is the reverse survivorship bias.

The reverse survivorship bias arises from the correlation between risk-adjusted returns and fund survival probabilities. It does not matter whether or not this correlation is the product of an underlying learning process about fund alphas. To appreciate this bias as a statistical result, suppose that a mutual fund has a fixed alpha $\alpha$ and that its risk-adjusted returns are $\tilde{R}_t^c \equiv \alpha + \tilde{\varepsilon}_t$, where $\tilde{\varepsilon}_t$’s have zero means and are independently distributed. A fund survives

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1See, for example, Brown, Goetzmann, Ibbotson, and Ross (1992), Elton, Gruber, and Blake (1996), Carpenter and Lynch (1999), and Carhart, Carpenter, Lynch, and Musto (2002).

2Pástor, Taylor, and Veronesi (2009) present a closely related argument in the IPO literature. They note that if a firm has an IPO after the posterior mean about the firm’s profitability exceeds some threshold, then the observed pre-IPO profitability must have been strictly higher than this threshold. As a consequence, the market rationally expects the firm to experience a post-IPO drop in profitability that is equal in size to the gap between the average pre-IPO profitability and the posterior mean at the time of the IPO.
for a random \( \tilde{T} \) number of months. The optional stopping-time theorem\(^3\) states that

\[
E\left[ \sum_{t=1}^{\tilde{T}} \tilde{\varepsilon}_t \right] = 0. \tag{1}
\]

It follows immediately from expression (1) that the expected average risk-adjusted return is

\[
E\left[ \frac{1}{\tilde{T}} \sum_{t=1}^{\tilde{T}} \tilde{R}^e_t \right] = \alpha + \text{cov}\left( \frac{1}{\tilde{T}}, \sum_{t=1}^{\tilde{T}} \tilde{\varepsilon}_t \right). \tag{2}
\]

This expression shows that if the survival probability is increasing in the risk-adjusted return, i.e., \( \text{corr}(\tilde{T}, \tilde{\varepsilon}_t) > 0 \), then the average risk-adjusted return is a downwards-biased measure of the fund’s true alpha, \( E\left[ \frac{1}{\tilde{T}} \sum_{t=1}^{\tilde{T}} \tilde{R}^e_t \right] < \alpha \).

Intuitively, this result arises from the ambiguity about whether a realized return is low because the true alpha is low or because the idiosyncratic shock is low. (It is this same noise that leaves a Bayesian investor’s posterior mean between the prior mean and the signal.) A fund manager may have a small positive alpha but a negative idiosyncratic shock results in a low return. If such a fund dies, it leaves behind an alpha estimate that is too low. There is no mechanism that would eliminate just-lucky mutual funds to offset this bias. I note that this bias cannot be resolved by studying mutual funds in isolation of each other. The covariance term in expression (2), which represents the bias, can only be addressed by studying the cross section of mutual funds.

The reverse survivorship bias argument is counterintuitive in its simplicity. I note that this bias is related to, but not an example of, the “baby-boy fallacy”: there will not be any more boys than girls if parents stop having children after having their first son. The difference between this fallacy and the reverse survivorship bias is the same as the difference between the sums and averages in expressions (1) and (2): do we count the total number of boys and girls in the population, or do we compute the average fraction of boys in each family? If parents were to follow a stop-at-a-boy rule, the boy and girl counts would be the same in the population but the fraction of girls in the average family would be just 0.31.\(^4\)

\(^3\)See, for example, Williams (1991).

\(^4\)Suppose that boys and girls are born with equal probabilities. The first born is a boy with probability \( \frac{1}{2} \). In
In this paper I first investigate the reverse survivorship bias by examining a Bayesian portfolio choice model that endogenizes mutual fund survival. I assume that a representative mutual fund investor can invest in a risk-free asset, the market portfolio, and an actively managed mutual fund. The key features of the model are that, first, the investor is uncertain about the fund’s alpha and, second, that the fund’s survival depends on the investor’s continuing investment. Each period the investor can abandon the existing fund and, if the investor exercises this option, the old mutual fund disappears and the investor draws a new fund with an unknown alpha. I find that, for reasonable parameter choices, the bias in the mean of the observed alpha distribution is about 1% per year.

I then construct and estimate a structural version of this model from the data. Similar to the portfolio choice model, I assume that each mutual fund’s alpha is fixed and drawn from a distribution that is known to investors. Each fund generates monthly risk-adjusted returns that exhibit two sources of uncertainty: idiosyncratic shocks to portfolio returns and the estimation uncertainty from the asset pricing model. Each period investors use Bayes’ rule to combine the prior distribution with the monthly risk-adjusted return to arrive at a posterior distribution for a fund’s alpha. Unlike in the portfolio choice model, I here remain agnostic about the attrition mechanism and model the fund disappearance probability as a free function of the posterior distribution. This function nests, as a special case, the possibility that a fund disappears when the posterior mean falls below some critical threshold. However, the structural model also can undo the built-in learning process if the data suggest that such a reversal is warranted.

I estimate the shape of the alpha distribution, the total variance of risk-adjusted returns, and the exit-probability function from the CRSP mutual fund data by using the Simulated Method of Moments. I match, between the data and simulations, the average alphas of both surviving and disappearing funds, the changes in these averages over time, the ten-year mutual fund survival rate, and the mean and the variance of the observed distribution of alphas. The model matches these salient characteristics of the mutual fund data. The estimated form such an event, the family stops and the fraction of girls in the family is zero. If the first born is a girl, then the family has a second baby and the probability that this baby is a boy is again $\frac{1}{2}$. The overall probability of such an event is $(\frac{1}{2})^2$ and the fraction of girls in the family is $\frac{1}{2}$. Continuing \textit{ad infinitum}, the expected fraction of girls in a family is then $\lim_{n \to \infty} \left\{ (\frac{1}{2})^1 \frac{1}{2} + (\frac{1}{2})^2 \frac{1}{2} + \cdots + (\frac{2^{n+1}}{2})^n \frac{1}{2} \right\} = \sum_{n=1}^{\infty} (\frac{1}{2})^n \frac{1}{2^{n-1}} = 0.3069$. 


of the exit-probability function supports the learning mechanism: a mutual fund typically disappears when there is over three-fifths probability that a fund’s true alpha is lower than $-3.5\%$ per year.

The structural model estimates indicate that the reverse survivorship bias is economically very important. While the average true (CAPM) alpha at the estimated parameter values is $-0.44\%$ per year, the observed alpha is just $-1.17\%$ per year. The difference in these figures, 73 basis points, is the estimate of the magnitude of the reverse survivorship bias. When the structural model is estimating by using the three- and four-factor model alphas, the magnitude of the bias is 104 and 88 basis points per year, respectively. These estimates of the size of the reverse survivorship bias, about 1% per year, are similar in magnitude to the estimates of the direct survivorship bias that plagued early mutual fund databases.\textsuperscript{5} These computations suggest that if we take a database that omits all dead funds and then start adding them back in, the (positive) direct survivorship bias decreases but, at the same time, the (negative) reverse survivorship bias begins to drag fund-specific alpha estimates down. When all dead funds have been added back in, as is done in the survivorship-bias-free databases, then the mean alpha estimate is too low by approximately 1% per year.

Although the model suggests that some mutual funds have positive (true) alphas net of fees, it also presents a gloomy picture of investors’ abilities to discover such funds. For example, the top 1% of fund managers have four-factor model alphas greater than 2.2% per year. It is the interaction between the noise in estimated alphas and the disappearance of some good funds by bad luck that impedes the discovery of positive-alpha funds. For example, if the simulated investors are handed the average $\alpha$-greater-than-1% fund and the attrition mechanism is shut down, it takes approximately twenty years for them to reject (in one-sided test) the null hypothesis that the true alpha is negative. When the mutual fund attrition is turned back on and investors have to restart the learning process each time a fund disappears, the average time to rejection increases by a factor of three. The benefit of carrying out these computations within the structural model is that the size of the test is precisely correct.

\textsuperscript{5}Grinblatt and Titman (1989) estimate that the direct (upwards) survivorship bias is “relatively small” and between 10 and 30 basis points per year; Brown and Goetzmann (1995) get estimates between 20 and 80 basis points; Elton, Gruber, and Blake (1996) find estimates between 71 and 91 basis points based on the three-factor model alphas; and Carhart, Carpenter, Lynch, and Musto (2002) show that the bias can be as large as 1% in samples longer than 15 years.
In reality econometricians (and investors) have to resolve the multiple-comparisons problem and correct for the fund return covariance structure in order to construct correctly sized test statistics.

The reverse survivorship bias affects the measurement of fund managers’ true alphas but it does not bias the estimates of the returns available to mutual fund investors. For example, the average return on a strategy that invests the same amount into each actively managed fund is unaffected because the profits of such a strategy do not depend on the counterfactual of how well a mutual fund would have performed had it not disappeared. The reverse survivorship bias is, however, of crucial importance if we are to draw inferences about fund managers’ stock-picking abilities: How many fund managers have positive alphas or what is the average fund manager’s alpha? The answers to these questions are important for understanding whether any active fund managers have access to valuable information. By contrast, the average return available to mutual fund investors, while an important statistic in its own right, does not measure heterogeneity in managers’ access to information.

Several recent studies have focused on the question of whether any fund managers are skilled, or whether all seemingly superior performance can be attributed to luck. Kosowski, Timmermann, Wermers, and White (2006) use a bootstrap technique and find that a “sizable minority” of managers pick stocks well enough to more than cover their costs. Barras, Scaillet, and Wermers (2009) examine the distribution of alphas (and the associated p-values) and find that the number of negative-alpha funds (24%) far outweighs the number of positive-alpha funds (0.6%). Fama and French (2009) extract demeaned risk-adjusted returns and run simulations to examine whether chance alone could generate the alpha distribution observed in the data. They find only weak evidence in net returns of some managers having enough skill to cover the costs they impose on investors. By contrast, they note that the left tail of the actual alpha distribution is far thicker than the tail of the simulated distribution. This left tail could indicate that some funds have either negative stock-picking skills or high trading costs, or both. I note that this finding also is consistent with the reverse survivorship bias. The alpha estimates are often too low for funds that disappear and so they thicken the left tail of the distribution.
I do not consider the equilibrium effects of money flows on alphas. In Berk and Green (2004) alphas get pushed to zero because investors can perfectly diversify all non-market risk and because the model’s mutual fund can monopolistically set its fee (to choose its own size). In Pástor and Stambaugh (2009), by contrast, equilibrium alphas do not equate to zero because, first, investors cannot diversify away all risk; second, because the active management industry is competitive; and third, because there is a finite number of mutual fund investors. I note that even if expected alphas are sensitive to money flows, the equilibrium considerations should not alter the premise of the reverse survivorship bias. As long as poorly performing mutual funds still shut down, as they do in the data, then the observed alphas will deviate from the true, unobserved alphas.

There is no simple resolution to the reverse survivorship bias because it is driven by the endogeneity between fund performance and survival and not by the lack of data. One cannot, for example, resolve this bias by applying a rule that leaves out funds that survive for fewer than $k$ years. Even if $k$ was known, such a rule could only get the mean of the distribution right by striking a balance between the direct and reverse survivorship biases. The overall shape of the observed alpha distribution would still differ from the true distribution and any inferences drawn about the managers in the tails would not be reliable.

This bias has significant implications beyond the evaluation of the abilities of mutual fund managers. The exit decisions of individual investors, for example, are very sensitive to poor investment performance. These investors usually stop trading after their trades lose money. My results on the economic significance of the reverse survivorship bias in the mutual fund data suggest that also individual investors’ alphas are not necessarily as low as what their realized returns suggest.

The paper is organized as follows. Section I describes the data and reports on the correlations between mutual fund survival and alternative alpha estimates. Section II calibrates the portfolio choice model with endogenous fund attrition to examine the reverse survivorship bias theoretically. Section III formulates the learning-based structural model, estimates it by using the simulated method of moments, and uses the estimated model to draw inferences about

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6See, for example, Seru, Shumway, and Stoffman (2009).
the size of the reverse survivorship bias and about the difficulty of discovering positive-alpha funds. Section IV concludes.

I. Data and Performance Measurement

I use the mutual fund data from the CRSP (Center for Research in Security Prices) database. I follow French (2008) and Fama and French (2009) and, first, include only funds that invest in U.S. common stocks and, second, combine different share classes of the same fund into a single fund. I restrict the sample to mutual funds that start on January 1984 or after this date. Although the CRSP mutual fund data start in 1962, the pre-1984 part of the data is not reliable. Fama and French (2009) also cut out the pre-1984 part of the data and note that the average returns in this part of the data are significantly higher for mutual funds that report monthly returns than for those that report annual returns.

I let a mutual fund enter the sample after its combined net asset value across all share classes exceeds $5 million in December 2006 dollars. Once a fund has exceeded this threshold, I keep the fund in the sample no matter what happens to avoid introducing a selection bias. This net-asset-value screen guards against the incubation bias of Evans (2009). The CRSP mutual fund files contain monthly returns up to the end of May 2009. I use returns up to April 2009 to be able to observe which funds are no longer active in May 2009. My sample contains data on 4,308 mutual funds that have return data for at least six months.

A. On the Correlation Between Alphas and Fund Disappearance

In this section I examine the correlation between fund returns and disappearance because of its importance in determining the size of the reverse survivorship bias (see expression 7).

7See, for example, Elton, Gruber, and Blake (2001) for a critical assessment of the accuracy of the pre-1984 segment of the CRSP Mutual Fund Database.

8Incubation bias arises from fund management companies providing seed money to new funds (to develop a return history) and then selectively opening the funds with best histories to the public. When these funds were allowed to backfill their return histories, such incubation drove a positive wedge between the fund’s past performance, which was selected to be good, and its expected performance. Fama and French (2009) use the same 5M threshold as their main specification and suggest that this limit probably exceeds the fund’s seed money and thus cuts out the pre-release period returns.
Although a number of studies\footnote{See, for example, Brown and Goetzmann (1995), Khorana (1996, 2001), Chevalier and Ellison (1999), and Lynch and Musto (2003).} find a positive relation between past returns and fund disappearance, I report on these relations because, first, my sample period (and sample construction) differs from those used in earlier studies, second, to show that data simulated from the portfolio choice model (in Section II) produces qualitatively relations, and third, because some of these estimates serve as inputs for the structural model estimation.

I use three measures of performance throughout this study: the CAPM alpha, the alpha from the three-factor model of Fama and French (1993), and the alpha from the four-factor model of Carhart (1997). I measure these alphas by employing time-series regressions such as

\[
R_{i,t} - r_{f,t} = \alpha_i + b_i (R_{m,t} - r_{f,t}) + s_i \text{SMB}_t + h_i \text{HML}_t + m_i \text{MOM}_t + e_{i,t},
\]

where \(R_{i,t}\) is the return on fund \(i\) in month \(t\), \(r_{f,t}\) is the risk-free rate, \(R_{m,t}\) is month-\(t\) return on the value-weighted CRSP index, and SMB\(_t\), HML\(_t\), and MOM\(_t\) are month-\(t\) returns on long-short portfolios for size, value, and momentum. I require that a fund has at least six months of return data to estimate its alpha. I update alphas continuously as funds age to measure the covariance between realized alphas and fund survival.

Table I reports on average alphas (as percentage points per year) for mutual funds that either survive through the \(t^{th}\) year or that perish in year \(t\). A mutual fund is included in year \(t\) analysis if the fund is still alive at the end of year \(t - 1\). If the fund perishes by the end of year \(t\), I compute the fund alpha by using all available data from inception until the last month of returns. If the fund is still alive at the end of year \(t\), I compute the alpha by using data up to the end of year \(t\). Each column in Table I reports on average alphas for funds that either survive or perish in year \(t\). For example, the four-factor model alphas indicate that the average alpha is \(-2.6\%\) per year for a fund that perishes at some point during its fifth year. By contrast, the average alpha is \(-0.15\%\) for a fund that survives through its fifth year.

The last row in the table, which reports the fraction of funds that survives at least \(t\) years, punctuates the economic importance of the disappearance of mutual funds. For example, approximately 10% of new mutual funds perish during the first three years of their lives and...
two thirds of mutual funds survive through the tenth year. (If a mutual fund is $T$-years old at the end of the CRSP mutual fund data set, I count the fund as a survivor in year 1, $\ldots$, $T$ computations but ignore it in year $T+1$, $\ldots$, 10 numbers.)

The alpha estimates suggest that mutual funds that perish perform considerably worse than surviving mutual funds. This conclusion is not sensitive to the year in which the comparison is made or to the choice of the asset pricing model that is used to estimate alphas. However, the size of the gap between the dead and surviving funds varies across the models. These differences in average alphas suggest that the disappearance of poorly performing funds is potentially a significant factor in performance evaluation via the reverse survivorship bias channel. For example, the reverse survivorship bias argument suggests that the three-factor model alpha estimate of $-7.3\%$ for those funds that survive for less than a year is most likely too low relative to these fund managers’ true abilities. The heart of the issue is that these realized alphas may be low not only because the true alpha is low but also because these funds experienced negative idiosyncratic shocks. Because of the latter possibility, the expected alpha for these funds at the time of disappearance was probably higher than $-7.3\%$ per year.

Figure 1 plots cumulative risk-adjusted returns for surviving and perishing funds to show how these returns vary from year to year leading up fund disappearance (or survival). In this figure I compute the risk-adjusted returns by using both the CAPM and the four-factor model. (I note that the three-factor model estimates are very similar to the four-factor model estimates and thus not reported.) The month-$t$ risk-adjusted return from the four-factor model is

$$
\hat{R}_{i,t}^e = R_{i,t} - r_{f,t} - \hat{b}_i (R_{m,t} - r_{f,t}) - \hat{s}_i \text{SMB}_t - \hat{h}_i \text{HML}_t - \hat{m}_i \text{MOM}_t,
$$

(4)

where $\hat{b}_i$, $\hat{s}_i$, $\hat{h}_i$, and $\hat{m}_i$ are fund $i$’s loadings on the market, size, value, and momentum factors. I estimate these loadings by using a time-series regression. If a fund perishes in year $t$, I again estimate its factor loadings by using all data up to the month of disappearance. If the fund survives, I use all data up to the end of year $t$. Figure 1 reports on cumulative risk-adjusted returns for funds that survive for at least two, four, six, eight, or ten years, as well as for funds that perish in years two, four, six, eight, or ten. The estimates suggest that mutual funds often disappear following a string of low returns. For example, mutual funds that perish in
their sixth year cumulatively lose close to 15% on a risk-adjusted basis in both specifications. While the performance of these funds is flat over the first two years, the subsequent four years of returns are consistently negative.

Table II reports on a set of probit regressions to measure the strength of the correlation between alpha estimates and mutual fund survival. In each regression the dependent variable takes the value of one if the fund disappears in year \( t \) and zero otherwise. I estimate these regression month by month, based on how long each fund has been in existence. For example, one regression uses data on funds that have existed for eight years and two months. I drop those months from the analysis in which no mutual fund disappears. I note the dependent variables in these cross-sectional regressions are independent of each other because for a fund to appear in month \( t \) regression, its dependent variable must have been zero in all previous regressions.

The regressor in Panel A is the fund’s alpha, estimated by using monthly returns up to month of the cross-sectional regression. Panel B reports on otherwise identical regressions but lags alpha estimates by a year. I again estimate alphas from the CAPM, the three-factor model, and the four-factor model.\(^{10}\) Table II reports on the average coefficient estimates across the monthly cross-sectional regressions.

The estimates in Table II support the same conclusion as the averages in Table I and the cumulative risk-adjusted returns in Figure 1. The probability that a mutual fund disappears from the data decreases significantly as the fund’s alpha estimate increases. The estimates for the CAPM regression in Panel A shows that the annualized exit probability is 0.04 for a fund with \( \hat{\alpha} = 0 \). This probability decreases close to zero (0.002) if the alpha estimate is 2% instead. By contrast, if the alpha estimate is \(-2\%\), the exit probability jumps up to 0.335.\(^{11}\) Thus, while the exit probability is close to zero for slightly negative alphas and for all positive

\(^{10}\) These regressions are not free of the errors-in-variables problem because the regressor is a first-stage alpha estimate. However, if these first-stage residuals are uncorrelated with fund survival, this errors-in-variables problem does not bias the estimates but lowers power by adding noise. Moreover, the average alphas reported in Table I show that the errors-in-variables problem should not materially influence the inferences. These Table I averages could be estimated from a similar survival regression but with its sides reversed: the alpha estimate would be the dependent variable and the regressors would represent interactions between years and survival.

\(^{11}\) I compute the annualized numbers as follows. If the alpha estimate is \(-2\%\), then the monthly exit probability, computed from the average point estimates reported in in Table II, is \( \Phi((-2.71) + (-0.02) \times (-43.88)) = 0.0334 \). Thus, the fund disappearance probability is 0.0344 in month 1, \((1 - 0.0344)(0.0344) = 0.0332\) in month 2, and so forth. The annualized exit probability is the sum of these projected exit probabilities that assume that the alpha estimate remains unchanged over the year.
alphas, it increases rapidly as the alpha estimate falls. The estimates in Panel B show that even if I lag alpha estimates by one year, realized alphas correlate significantly with mutual fund disappearance.

The probit regression estimates are very similar across the three asset pricing models. Two effects may make it difficult to distinguish between these models in exit regressions. First, the more complicated the asset-pricing model is, the more data is needed to estimate alphas with the same precision. If the amount of data is fixed, then the addition of each new factor decreases the precision at which alphas are estimated. For example, even if it were the four-factor model alphas that correlated perfectly with survival, the noisiness of these alphas in the data relative to the CAPM alphas could tilt the regressions to favor the CAPM. Second, if fund disappearance is related to investors’ inferences about alphas, the relevant question is not what is the true or ideal asset pricing model but what is the asset pricing model used by the investors. For example, before Carhart (1997), the momentum portfolio was not typically included as a risk factor (or a passive benchmark) in mutual fund performance studies.12

The estimates in Tables I and II and in Figure 1 indicate that fund survival correlates significantly with mutual fund performance when performance is measured by fund alphas. These findings, which are consistent with the prior literature on the (direct) survivorship bias, suggest that the reverse survivorship bias will affect inferences about the distribution of fund managers’ abilities.

II. A Bayesian Portfolio Choice Model with Endogenous Fund Attrition

A. Assumptions

I solve a portfolio choice problem in which a representative mutual fund investor can invest in a risk-free asset, the market portfolio, and an actively managed mutual fund. The key features of the model are that, first, the investor is uncertain about the fund’s alpha and,

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12Cremers, Petäjistö, and Zitzewitz (2008) note that practitioners commonly evaluate fund managers by comparing their returns to benchmark indices, such as the S&P 500 for large-cap stocks and Russell 2000 for small-cap stocks. No direct adjustment is commonly made for funds’ exposure to the momentum factor.
second, that the fund’s survival depends on the investor’s continuing investment. Each period
the investor can abandon the current fund and, if the investor exercises this option, then the
old mutual fund disappears and the investor draws a new fund with an unknown alpha. This
switch “resets” the investor’s prior belief about the fund’s alpha. I calibrate this model, first,
to examine the (possible) economic significance of the reverse survivorship bias and, second,
to study the rule that the investor uses to decide when to exercise the abandonment option
and how the time-series changes in this rule interact with the bias.\(^\text{13}\)

I assume that an infinite-horizon investor maximizes log-utility over consumption:

\[
E \left[ \sum_{t=0}^{\infty} \beta^t \log c_t \right],
\]  

where \( \beta \) is the investor’s discount rate. The wealth dynamics are given by

\[
W_{t+1} = (W_t - c_t) \left( 1 + r_f + \theta_m (\bar{r}_{m,t} - r_f) + \theta_z (\bar{z}_t - r_f) \right),
\]  

where \( r_f \) is the risk-free rate, \( \theta_m \) is the proportion of wealth invested in the market portfolio,
\( \bar{r}_{m,t} \) is the return on the market portfolio, \( \theta_z \) is the proportion of wealth invested in the mutual
fund, and \( \bar{z}_t \) is the return on the mutual fund. I assume that \( \theta_z \geq \bar{\theta} \) to impose a (possibly
small) minimum investment size. The investor knows all parameters of the problem except
for the mean of \( \bar{z}_t \). The date-\( t \) return on the market portfolio is

\[
\bar{r}_{m,t} = \mu_m + \bar{\epsilon}_{m,t},
\]  

where \( \mu_m \) is a known constant and \( \bar{\epsilon}_{m,t} \)’s are i.i.d. from period to period. I assume that \( \bar{\epsilon}_{m,t} \)
is drawn from a left-truncated normal distribution with truncation at \( x = -1 \). The underlying
untruncated distribution has a mean of zero and a variance of \( \sigma^2_{m} \). I assume that the fund’s

\(^{13}\)Dangl, Wu, and Zechner (2008) also study a model in which investors learn about mutual fund managers' abilities and the management company has the option replace the manager. Some of their results are similar to mine. For example, Dangl et al. find that the management company may fire a manager with an above-average alpha because a replacement (with an unknown alpha) can have more upside potential. However, while they focus on the model’s implications on the relations between fund size, portfolio risk, and fund manager’s tenure, I examine how the endogenous survival mechanism biases fund-specific performance estimates. The mechanics of their model also are very different. They assume the Berk and Green (2004) equilibrium and thus do not consider the problem that fund investors face. By contrast, I start from these investors’ problem but, unlike Dangl et al., do not model the management company.

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market-model beta is one so that its return is
\[ \tilde{z}_t = \alpha + \tilde{r}_{m,t} + \tilde{\varepsilon}_{z,t}, \]  
(8)

where \( \alpha \) is not known to the investor (but he has a prior about it). I assume that also \( \tilde{\varepsilon}_{z,t}'s \) are drawn from a left-truncated normal distribution with truncation at \( x = -1 \). The underlying untruncated distribution has a mean of zero and a variance of \( \sigma^2_z \). I apply these truncation assumptions to give both the market portfolio and the mutual fund limited liability. The model that I have in mind is one where the fund's market exposure is unknown and the estimation of this exposure from the data increases the variance of signals about alpha. However, instead of formalizing this by modeling \( \beta \) as either unknown to the investor or just random i.i.d. draw each period, I interpret expression (8) as delegating, for simplicity, this estimation uncertainty component to the residual \( \tilde{\varepsilon}_{z,t} \).

The investor updates his beliefs after each date as follows. First, after observing both fund and market returns, the investor backs out the signal \( s_t \equiv \tilde{z}_t - \tilde{r}_{m,t} \) about the fund alpha. The investor then reverses the truncation by computing the signal realization \( s'_t \) by mapping the truncated distribution to an untruncated distribution. I compute this transformation by first looking up the cumulative density at \( s_t \) and then find the value \( s'_t \) in the untruncated distribution that has the same density. The investor's date-\( t \) prior belief about the mean of this untruncated distribution has a mean of \( m_t \) and a variance of \( v_t \). I assume that the initial prior distribution coincides with the true population distribution of alphas, \( N(\mu_\alpha, \sigma^2_\alpha) \). The dynamics of the mean and variance of the belief distribution are given by\(^{14}\)

\[ m_{t+1} = \frac{m_t}{v_t} + \frac{s'_t}{\sigma^2_z}, \]  
(9)

\[ v_{t+1} = \frac{1}{\frac{1}{v_t} + \frac{1}{\sigma^2_z}}, \]  
(10)

\(^{14}\)The trick that I apply here to keep the posterior distribution closed under updating is more transparent in an alternative setup with log-normally distributed returns. Instead of working with a log-normal likelihood function, an investor could be uncertain about the mean of the normal variate \( \tilde{x} \) in \( \tilde{r} \equiv e^{\tilde{x}} - 1 \). The investor would then back out from each return observation the normal variate realization, \( \log(1 + \tilde{r}) \). I do not use this log-normal assumption because of its downside that the variance of \( \tilde{r} \) also changes as the mean of \( \tilde{x} \) changes. By contrast, the truncation of the normal distribution at \( x = -1 \) is an innocuous return-distribution twist because, for reasonable return volatilities, the amount of truncated mass is effectively zero. See Johnson, Kotz, and Balakrishnan (1994) for details on truncated normal distributions.
Each period the investor can abandon the current fund (thus causing the fund to die) and, if
the investor so chooses, draw a new fund with an unknown alpha. I assume that abandoning
a fund and drawing a new funds costs the representative investor $\kappa$ percent of wealth. This
cost, which can be zero, is the strike price for the option of restarting the learning problem
with a new fund.

\section*{B. Solution}

The investor’s indirect utility is a function of three state variables: the current wealth $W_t$, the
mean of the prior distribution $m_t$, and the variance of the prior distribution, $v_t$. Because the
investor maximizes log-utility, the wealth and belief terms are additive in the indirect utility
function. I conjecture that

$$V(W_t, m_t, v_t) = A + B \log W_t + g(m_t, v_t), \quad (11)$$

where $A$ and $B$ are constants and $g(\cdot)$ is a function of the investor’s date-$t$ beliefs. The
investor’s optimization problem with this conjecture becomes

$$V(W_t, m_t, v_t) = \max_{c_t, \theta_m, \theta_z \geq \theta} \left\{ \log c_t + \beta V \left( (W_t - c_t) \left( 1 + r_f + \theta_m (\tilde{r}_{m,t} - r_f) + \theta_z (\tilde{z}_t - r_f) \right), \tilde{m}_{t+1}, \tilde{v}_{t+1} \right) \right\}$$

$$= \beta A + \max_{c_t} \left\{ \log c_t + \beta B \log (w_t - c_t) \right\}$$

$$+ \max_{\theta_m, \theta_z \geq \theta} \left\{ \beta B \max_{\theta_m} \left\{ \mathbb{E} \left[ \log \left( 1 + r_f + \theta_m (\tilde{r}_{m,t} - r_f) \right) \right] + \beta \mathbb{E}[g(m_t, v_t)] \right\} \right\}$$

$$+ \beta B \max_{\theta_m, \theta_z \geq \theta} \left\{ \mathbb{E} \left[ \log \left( 1 + r_f + \theta_m (\tilde{r}_{m,t} - r_f) + \theta_z (\tilde{z}_t - r_f) \right) \right] + \beta \mathbb{E}[g(\tilde{m}_{t+1}, v_{t+1})] \right\}$$

$$+ \beta B \max_{\theta_m, \theta_z \geq \theta} \left\{ \mathbb{E} \left[ \log \left( 1 + r_f + \theta_m (\tilde{r}_{m,t} - r_f) + \theta_z (\tilde{z}_t - r_f) - \kappa \right) \right] + \beta \mathbb{E}[g(\tilde{m}_1, v_1)] \right\}. \quad (12)$$

The last three lines of expression (12) capture the fact that each period an investor can
make a choice that disrupts the natural evolution of beliefs from $(m_t, v_t)$ to $(\tilde{m}_{t+1}, v_{t+1})$. The
first possibility, shown on third to the last line, is that the investor decides to invest in neither
the current nor a new mutual fund. The investor withdraws the money from current fund, the fund disappears, and the evolution of beliefs stops. I note that if $\kappa$ is low and if the investor has ever invested in a mutual fund, then the investor can never reach a belief state in which he would choose this option. If the investor has ever invested in a fund, then the option to draw a new fund must dominate this quitting choice. This option is chosen only if the distribution of alphas is so unattractive that the investor never invests in a mutual fund.

The second possibility, shown on second to the last line, is that the investor stays with the current mutual fund. The investor’s belief about the mean mutual fund return $\tilde{z}_t$ is $m_t$ and the investor updates his beliefs to $(\tilde{m}_{t+1}, v_{t+1})$ after observing the risk-adjusted (and transformed) return. The third possibility, shown on the last line, is that the investor abandons the current mutual fund (and the fund disappears) and draws a new fund. The investor’s prior distribution about the new fund is the same as the original prior distribution, $(m_0, v_0)$, so the investor’s beliefs “restart” if the investor exercises this abandonment option. If the investor switches to a new fund, the investor’s prior mean about the next-period mutual fund return is $m_0$ and the investor updates to $(\tilde{m}_1, v_1)$ based on the return realization. The investor pays $\kappa$ to exercise this option.

The optimal consumption from the first-order condition of expression (12) is $c^*_t = \frac{1}{1+\beta B} W_t$. Constants $A$ and $B$ can be solved by inserting the optimal consumption back into expression (12) and matching the coefficients against the value-function conjecture from expression (11). The value function then simplifies to

$$V(W_t, m_t, v_t) = \frac{\beta \log \beta + (1 - \beta) \log (1 - \beta)}{(1 - \beta)^2} + \frac{1}{1 - \beta} \log W_t + g(m_t, v_t), \quad (13)$$
where $g(m_t, v_t)$ solves the following functional equation:

\[
g(m_t, v_t) = \beta \max_{\theta_m} \left\{ \frac{1}{\beta(1-\beta)} \max_{\theta_z} \left\{ \mathbb{E} \left[ \log \left( 1 + r_f + \theta_m (\tilde{r}_{m,t} - r_f) \right) \right] \right\}, \right. \\
+ \frac{1}{1-\beta} \max_{\theta_m, \theta_z \geq \tilde{\theta}} \left\{ \mathbb{E} \left[ \log \left( 1 + r_f + \theta_m (\tilde{r}_{m,t} - r_f) + \theta_z (\tilde{z}_t - r_f) \right) \right] + \mathbb{E} [g(m_{t+1}, v_{t+1})], \right. \\
+ \frac{1}{1-\beta} \max_{\theta_m, \theta_z \geq \tilde{\theta}} \left\{ \mathbb{E} \left[ \log \left( 1 + r_f + \theta_m (\tilde{r}_{m,t} - r_f) + \theta_z (\tilde{z}_t - r_f) - \kappa \right) \right] + \mathbb{E} [g(m_{t+1}, v_{t+1})] \right\}. \\
\]  

(14)

Expression (13) verifies the conjecture about the form of the value function. The investor’s optimal investment decisions are determined by the two-state-variable function $g(m_t, v_t)$ in expression (14). The first line of expression (14) is a simplified version of the third line of expression (12). This simplification follows from noting that if an investor ever abandons a mutual fund without picking a new one, then the investor’s beliefs must forever remain stuck in the same state. The investor’s problem then simplifies to a standard stochastic log-utility investment problem which has the solution shown on the first line of expression (13). I note that the function $g(m_t, v_t)$ also can be solved (up to a static portfolio choice problem) in those belief states where the investor has resolved all uncertainty about the fund alpha. Analogously to the permanent-exit case, such an investor must remain in the same belief state, and the value of $g(m_t, v_t)$ at such a $v_t = 0$ boundary point is

\[
g(m_t, 0) = \frac{\beta}{(1-\beta)^2} \max_{\theta_m, \theta_z \geq \tilde{\theta}} \left\{ \mathbb{E} \left[ \log \left( 1 + r_f + \theta_m (\tilde{r}_{m,t} - r_f) + \theta_z (\tilde{z}_t - r_f) \right) \right] \right\}. \\
\]  

(15)

The form of function $g(m_t, v_t)$ in expression (14) is informative about the investor’s behavior in this model. An investor’s decision to either remain with the current fund or to switch to a new fund depends on the future evolution of beliefs. The evolution of beliefs, in turn, depends crucially on the variance of beliefs. If the prior distribution about a fresh mutual fund’s alpha “dominates” the posterior distribution about the current fund’s alpha (after paying the strike price $\kappa$), then the investor abandons the current fund and restarts the problem. The log-utility assumption, which shuts down the intertemporal hedging demand
component, cleanly isolates the value of this abandonment option. Moreover, because of the same log-utility assumption, the date-\(t\) portfolio choice problems in expression (14) are all static and can be solved separately before starting the search for the solution of \(g(m_t, v_t)\).

I solve the problem in three steps. First, I create a mean-variance belief grid for \(m_t\) and \(v_t\) with 800 points for \(m_t\) and 1,200 points for \(v_t\).\(^{15}\) Second, I solve the static portfolio choice problems in expression (14) for each grid point. Third, I generate initial guesses about the value of \(g(m_t, v_t)\) for each grid point and then start iterating over the grid, sweeping recursively from \(v_T\) towards \(v_0\) at each iteration. I compute the value of \(g(m_t, v_t)\) from expression (14) at each grid point given the initial guesses of the value of this function at other grid points. I note that the expectation about next period’s \(g(\tilde{m}_{t+1}, v_{t+1})\) depend on the uncertain evolution of beliefs. I compute the transition probabilities from \(m_t\) to all different states \(\tilde{m}_{t+1}\) using expression (9) together with the distributional assumptions about the signal \(\tilde{s}_t\). I iterate over the mean-variance belief grid until the values of \(g(m_t, v_t)\) have converged at each grid point. The solution to the investor’s problem requires value-function iterations because of the abandonment option. An investor who abandons the fund transitions back to \(g(m_0, v_0)\) and the function value in this grid point, in turn, depends on the investor’s optimal choices in every possible state that follows.

\(\textbf{C. Calibration}\)

I fix the parameters of the model to the following (annualized) values. First, I assume that each period in the model represents one month. I set the mean of the true alpha distribution \((\mu_\alpha)\) to \(-0.5\%\) and its standard deviation \((\sigma_\alpha)\) to 0.682\%. These parameter values imply that about 10\% of funds have (true) alphas greater than 2.5\% per year and that 1\% of all managers have alphas above 5\% per year. Second, I fix the non-mutual fund parameters of the model by setting the risk-free rate \((r_f)\) to 5\%, the expected return on the market portfolio \((\mu_m)\) to 10\%, and the standard deviation of this return \((\sigma_m)\) to 30\%. I set the investor’s discount rate \((\beta)\) to 0.95.

\(^{15}\)I choose the grid for \(v_t\) to match the deterministic evolution of the posterior variance in expression (10). Thus, the last node corresponds to the variance of beliefs after having invested in the same fund for 100 years. (I calibrate the model so that one period corresponds to one month.) I assume that the variance of beliefs drops to zero after this date. I create the grid for \(m_t\) so that it covers 99.95\% of the true population distribution of \(\alpha\).
Panel A of Table III reports on ten-year survival rates in the model based on different choices for the remaining three fund-specific parameters. I let the minimum mutual fund investment size to take values $\theta = 0, 0.01, \text{ or } 0.02$. I use these same parameter values for the strike price $\kappa$, i.e., for the cost of abandoning a fund. Finally, I select the variance of risk-adjusted fund returns ($\sigma^2_z$) based on what the parameter value choices imply about the speed of learning. I consider three different parameter values so that the variance of posterior distribution halves every second year ($\sigma_z = 3.34\% \text{ per year}$), every fifth year ($\sigma_z = 5.29\% \text{ per year}$), or every tenth year ($\sigma_z = 7.48\% \text{ per year}$). I note that the representative fund investor’s initial 95% confidence interval about the annualized alpha is $(-5.13\%, 4.13\%)$. If the mean remains unchanged but the variance halves, then this 95% confidence interval narrows to $(-3.77\%, 2.77\%)$.

The survival rates in Panel A suggest that the mutual fund investor abandons funds far too often, relative to actual data, if the cost of switching a fund is set to zero. Whereas the actual ten-year survival rate in the data is two thirds, only approximately 5% of funds survive in the model up to ten years when $\kappa$ is zero. The reason for this low survival rate is that, without a switching cost, almost any negative return in the very beginning turns the current fund’s posterior distribution inferior relative to a new fund’s posterior distribution. An increase in the cost of abandoning a fund to 0.01 increases survival rates because it forces the investor to weigh this cost against the benefits of switching.

If the investor can learn about alphas too fast, then the model-based survival rates are again too low, less than 50% over the first ten years. The five-year (middle) choice for the speed of learning (i.e., for the time it takes to halve the variance), by contrast, yields more realistic survival rates. For example, when the speed of learning is set to five years, the cost of abandonment is 0.01, and the minimum investment size is 0.01, then the survival rate in the model is approximately two fifths, which is quite close to the survival rate in the data. In the following analysis I use this set of parameter values to further study the model. I also include another specification that sets the cost $\kappa$ to 0.0125. This alternative choice has a ten-year survival rate of 65.2% which is almost the same as the survival rate in the data.

Panel B of Table III reports on the average alphas (percentage points per year) for mutual
funds that either survive through the $t^{th}$ year or that perish in year $t$. Both the underlying computations and the presentation of the results in this table are identical to Table I, which is based on the CRSP sample. The alpha estimates in the model-based simulations exhibit the same upward-sloping pattern that is observed in the actual sample. In both specifications, the average alpha is negative for both surviving and dead funds in year one (0.95\% and −18.97\%, respectively), but both averages increase over time. For example, the average observed alpha is 1.35\% per year in the first specification for funds that survive through the tenth year. By contrast, the average alpha is −1.99\% for funds that disappear during the tenth year.

Figure 2 plots the critical posterior mean value in the model that the investor uses to decide whether or not to abandon the existing fund and to draw a replacement fund from the true alpha distribution. The critical threshold is initially approximately −3.5\% with $\kappa = 0.01$ and −4.0\% with $\kappa = 0.0125$. These low threshold values suggest that the investor abandons a fund early only if the realized return is very low. This low threshold value thus explains why alphas of the first-year dead are as low as they are in Panel B. These low realizations are required to drag the posterior mean below −3.5\%. I note that the gap between this realized value and the actual posterior mean at the time of the disappearance, which is always higher and closer to the quitting threshold, represents the reverse survivorship bias. Figure 2 shows that the critical alpha threshold increases smoothly over time. After ten years, the critical value is approximately −1.4\% in the first specification and after twenty five years, the threshold is −0.26\%. Thus, similar to Dangl, Wu, and Zechner (2008), the representative fund investor eventually abandons a fund with an above-average alpha because, after 25 years, a new fund manager has more upside potential. The increase in the critical threshold leads to the increasing pattern in average alphas that is seen in Panel B of Table III.

I note that the investor may invest in a fund at date zero even if the prior belief about its alpha is very low. The investor is willing to give the fund manager a chance because the alpha could be positive. If the fund posts low returns, the investor eventually abandons the fund in favor of a new fund, and the old fund disappears. If the fund posts high returns, the investor infers skill and keeps investing in the fund. By contrast, in the model of Dangl, Wu, and Zechner (2008) a fund can never exist if the market’s prior mean about the manager’s alpha is negative. The reason is that their model builds on top of the Berk and Green (2004) model in
which fund sizes are assumed to equate the net of fees alpha to zero. Such equilibrium cannot be supported in the negative-alpha region with positive fund fees.

Panel C of Table III reports on the size of the reverse survivorship bias within this portfolio choice model. I simulate at most 25 years of monthly return data for each fund to match the length of the CRSP data. Although the true mean of the alpha distribution is $-0.5\%$ in the calibrated model, the observed distributions have significantly lower means because of attrition. In the first specification, the observed alpha is $1.1\%$ lower than the true mean and in the second specification this gap is $0.81\%$. The percentiles for the observed and true alpha distributions show that the reverse survivorship bias does not evenly shift the distribution downwards. The funds that disappear must have low realized alphas, so the salient result of the reverse survivorship bias is the thickening the left tail of the observed alpha distribution relative to the the true distribution. The worst 5% of funds in the observed alpha distribution have alpha realizations that are $-8.9\%$ or lower. The true 5th percentile of the distribution, by contrast, is $-4.4\%$. These calibrations thus suggest that the reverse survivorship bias can significantly affect the shape of the observed alpha distribution. These calibrations also indicate that, for reasonable parameter choices, the shift in the mean can be very significant, around 1% per year.

III. Simulated Method of Moments Estimation

A. Structural Model

In this section I construct a structural model of mutual fund survival that I then estimate using the CRSP fund data. This model is based on the portfolio choice model except that the fund exit probability is a general function of the posterior distribution. Thus, whereas the exit function in the portfolio choice model follows from the assumption about the representative investor’s preferences (and from the assumption that each fund exists in isolation of other funds), I keep the structural model more general by not specifying the underlying attrition mechanism.

Similar to the portfolio choice model I assume that each mutual fund’s (unobserved) alpha is drawn from a normal distribution with a mean $\mu$ and variance $\sigma^2$. The market knows the
parameters of this distribution. I assume that investors cannot initially distinguish funds from each other so that this normal distribution is also the market’s prior distribution about each fund’s alpha. The prior distribution’s mean and variance at date zero are thus \( m_0 = \mu \) and \( v_0 = \sigma^2 \), respectively.

Each mutual fund generates monthly return observations. I assume that the market uses some asset pricing model to obtain an estimate of the fund’s monthly risk-adjusted return, \( \hat{R}_{i,t}^e \equiv \alpha_i + \tilde{\varepsilon}_{i,t} \), where \( \tilde{\varepsilon}_{i,t} \) is normally distributed with a mean zero and variance \( \sigma^2_e \). This risk-adjusted return variance reflects two sources of uncertainty: the idiosyncratic shocks to returns and the estimation uncertainty that arises from having to use a finite sample to estimate the asset pricing model for risk adjustment. I note that the market’s asset pricing model is correct in that \( E[\hat{R}_{i,t}^e] = \alpha_i \).

The market uses each risk-adjusted monthly return realization to update from the prior distribution to the posterior distribution. The mean and the variance of the posterior distribution are, similar to expressions (9) and (10),

\[
\begin{align*}
m_{i,t} &= \frac{m_{i,t-1}}{v_{t-1}} + \frac{\tilde{\varepsilon}_{i,t}}{\sigma^2_e}, \quad \text{and} \\
v_{t} &= \frac{1}{v_{t-1}} + \frac{1}{\sigma^2_e},
\end{align*}
\]

where the posterior variance does not have the fund subscript \( i \) because it follows a deterministic process. I assume that the probability that a fund disappears after month \( t \) is a function of the fund’s posterior distribution \( N(m_{i,t}, v_t) \). I construct this exit function as follows. First, let \( \bar{\alpha} \) denote some critical level of alpha. I then compute and record the probability that the fund alpha is below this level, \( p_{i,t} \equiv \int_{-\infty}^{\bar{\alpha}} \phi(x; m_{i,t}, v_t)dx \). I assume that a fund disappears from the data between months \( t \) and \( t + 1 \) with probability \( \xi(p_{i,t}, t) \), where \( \xi(\cdot) \) is a possibly non-linear and time-dependent function. These assumptions state that the probability that a fund disappears is a function of the amount of mass in the posterior distribution below some critical threshold \( \bar{\alpha} \). I expand \( \xi(\cdot) \) twice around \( p_{i,t} = 0 \) and allow it to depend linearly on the
fund’s age. The approximation of $\xi(\cdot)$ that I estimate from the data is then

$$\xi(p_{i,t}, t) \approx \hat{\xi}(p_{i,t}, t) = \max(\min(\gamma_0 + \gamma_1 p_{i,t} + \gamma_2 p_{i,t}^2 + \gamma_3 t, 1), 0),$$

(18)

where $t$ is the fund’s age in years and the minimum and maximum operators bound the probability between zero and one. I adopt this free function-based approach to remain agnostic about the mechanics of mutual fund attrition. When I estimate both the critical threshold and parameters of $\hat{\xi}(p_{i,t}, t)$ from the data, the structural model can “undo” the built-in learning process if the data suggest that such a reversal is warranted. However, I note that this five-parameter specification nests, as a special case, the possibility that a fund disappears when the posterior mean falls below some endogenous threshold. The estimates of $\bar{\alpha}$ and $\hat{\xi}(p_{i,t}, t)$ also are thus informative about whether mutual funds indeed disappear when their posterior distributions fall below some threshold, or if the disappearance is a more-arbitrary function of the posterior distribution.

The eight structural parameters of the problem that I estimate from the data are $\mu$ (the mean of the alpha distribution), $\sigma^2$ (the variance of the alpha distribution), $\sigma^2_e$ (the total variance of risk-adjusted returns), $\bar{\alpha}$ (the critical level in the posterior distribution computation), and $\gamma_0$, $\gamma_1$, $\gamma_2$, and $\gamma_3$ (the parameters of the function that translate the amount of probability mass below $\bar{\alpha}$ into an exit probability).

I use thirteen moment conditions to identify these eight structural parameters. The first five moment conditions represent the means of the alpha distributions for the funds that survive. These are the same numbers as the per-year averages reported on in Table I except that I aggregate the data to biannual frequency to reduce noise. The next five moment conditions represent the means of the alpha distributions for funds that perish in these same two-year periods. These ten moment conditions instruct the structural model to match the levels of alphas for surviving and perishing funds and to match the changes in these alphas as funds mature. The next moment condition is the ten-year survival rate, which ensures that the rate at which mutual funds fall out of the model matches the attrition rate observed in the data. The average and the variance of the overall alpha distribution constitute the last two moment conditions. I compute each fund’s alpha by using the data up to the last month
of returns, or up to the end of the fund’s tenth year (which ever comes sooner), to construct this overall alpha distribution. The inclusion of these moments instructs the model to push the observed alpha distribution in the model close to the alpha distribution observed in the data. I use these thirteen moments to estimate the eight structural parameters parameters \((\mu, \sigma^2, \sigma_e^2, \bar{\alpha}, \gamma_0, \gamma_1, \gamma_2, \gamma_3)\) to have an overidentified model.

Because the structural model does not yield closed-form estimation equations, I use the simulated method of moments (SMM) for indirect inference. I draw random funds from the true alpha distribution, generate monthly risk-adjusted returns, update the market’s beliefs using expressions (16) and (17), and then compute the probability that a mutual fund disappears from the data. I create new funds until I have generated a large simulated sample of mutual funds from which to compute the simulated moments. The SMM-estimator \(\hat{\theta}\) is then

\[
\hat{\theta} = \arg \min_{\theta} \left( \hat{M} - M(\theta) \right)^\prime W \left( \hat{M} - M(\theta) \right),
\]

where \(\theta\) is an eight-by-one vector of the structural parameters, \(\hat{M}\) is the vector of estimated moments from the data, \(M(\theta)\) is the vector of model-implied moments, and \(W\) is an arbitrary positive-definite weighting matrix. I use the optimal weighting matrix for estimation.

The downside of this simulation approach is that the simulated moments reflect randomness in the simulation process. Even if the number of simulated funds is very large, the resultant simulated moments are too noisy to use derivative-based methods to minimize expression (19). Instead of applying the simulated-annealing method, I exploit the specific structure of the problem to approximate the objective function in expression (19). Before I begin the minimization routine, I generate a set of primitive \(U(0, 1)\) random variables. If the simulations are for \(N\) funds, I generate \(N\) primitive draws of mutual fund alphas and \(N \times 120\) (i.e., ten years) primitive draws for monthly risk-adjusted returns. I then save these primitive random variables and, each time I re-evaluate expression (19), I recall this set of draws. For example, as the structural parameters \(\mu\) and \(\sigma^2\) change by a small amount, corresponding to a change in the shape of the alpha distribution, the same \(N\) primitive random variables now correspond to (slightly) different fund alphas. By using pre-generated random variables for fund alphas and returns, the objective function is now continuous in all structural parameters of the problem.
In addition to the use of pre-generated random variables, I also allow each mutual fund to both disappear and stay alive each month. (A more limited approach would test each month whether a mutual fund disappears or not, and when the fund disappears, record the history of the fund.) Each month I compute the probability that a mutual fund disappears and, based on these monthly exit probabilities, I also compute the probability that the fund is alive in months \( t = 2, 3, \ldots, 120 \). I thus use each simulated fund to construct \( 2 \times 120 \) fund observations (with different probability weights), where each observation corresponds to a fund either disappearing or staying alive in month \( t \). Whereas a simulated-annealing approach needs to keep \( N \) small (because the objective function has to be evaluated a large number of times), I can choose a large \( N \) so that the simulated moments inherit only a small amount of the randomness in the original draws of the primitive random variables. I set \( N = 250,000 \), so that the simulated sample is approximately fifty times as large as the actual mutual fund sample.

### B. Estimation Results

Table IV compares the moment conditions between the data and the structural model (Panel A) and reports on the estimates of the eight structural parameters (Panel B). The \( \chi^2 \) test statistic in Panel B indicates that the learning-based model has the best fit to the data when the alphas are estimated from the CAPM. This result may suggest that the market uses the CAPM (and not the three- or four-factor models) to evaluate fund performance and to determine which funds should disappear, or that the three- and four-factor models produce noisy alpha estimates relative to the CAPM. However, although the model is rejected under the three- and four-factor model alphas, I note that the inferences from the model are very similar irrespective of which asset pricing model is used to construct the sample moments.

The structural model matches the salient features of the mutual fund data. First, the observed alphas are very low for funds that disappear early on but then increase monotonically in the disappearance date. For example, the average CAPM alpha in the data is \(-5.9\%\) for funds that disappear within the first two years. The corresponding simulated number at the estimated parameter values is \(-4.4\%\). (The alphas in Table IV are reported as percentage points per month.) For funds that disappear during years nine and ten, these alphas are
2.1% and −1.8%, respectively. The CAPM alphas of surviving funds also increase in fund age both in the data and the simulations. It is these moment conditions, however, that cause difficulties for the model with the three- and four-factor model alphas. Unlike the CAPM alphas, the three- and four-factor alphas in the data start positive but then turn negative after five years. (The reversed U-shape pattern in the surviving four-factor model fund alphas is shown in Figure 1.) Although the model is partially successful in matching this pattern, the remaining discrepancies are sufficiently large to reject the model in the three- and four-factor specifications.

In contrast to the alphas of the surviving funds, the model matches each of the remaining moment conditions irrespective of which asset pricing model is used to measure alphas. The ten-year survival rate, for example, is nearly the same in each version of the estimated model as what it is in the data (66.6%). Although the mean of the overall alpha distribution is very different depending on which asset pricing model is used to compute these alphas, the model accommodates these shifts and also matches the variance of the alpha distribution.

The first two parameters in Panel B describe the shape of the true alpha distribution. (This distribution also constitutes mutual fund investors’ prior distribution for each fund alpha in the model.) While I report on the annualized parameters, I note that the fund alphas in the model are drawn from the underlying monthly distribution. Both the mean and the dispersion of alphas is significantly greater under the CAPM than what they are under the three- and four-factor models. For example, while 6% of funds have (true) annual alphas greater than 5% under the CAPM, this proportion is effectively zero for both the three- and four-factor models.

The risk-adjusted returns, which comprise both the idiosyncratic fund-return component and the estimation uncertainty inherited from the asset pricing model, have a monthly standard deviation of 1.11% under the CAPM. This estimate is quite high in terms of what it implies about the speed at which the market can resolve uncertainty about fund alphas. I note that in the underlying normal-normal updating problem, the variance of the date-
posterior, ignoring the effect of fund attrition, is

\[ v_T^2 = \frac{1}{v_0^2 + \sigma_e^2}. \tag{20} \]

This expression indicates that the time \( T \) that it takes to reach any given posterior variance \( v_T \) is

\[ T = \left( \frac{1}{v_T^2} - \frac{1}{v_0^2} \right) \sigma_e^2. \]

Given the CAPM estimates and the monthly learning frequency, the estimate of the variance of the risk-adjusted returns implies that it takes almost eleven years until the posterior variance drops to one tenth of the prior variance. This noisiness of risk-adjusted returns coupled with fund attrition, as discussed below, will significantly impair investors’ abilities to discover positive-alpha funds.

The critical alpha level, which investors in the model use to construct an input for the mutual fund exit rule, is \(-3.5\%\) (per year) for the CAPM and approximately \(-2.8\%\) for both the three- and four-factor models. (This critical alpha level, which is estimated quite precisely in each of the three models, gives the boundary that the market uses to construct the probability-mass input for the stochastic exit function.) Unlike for the critical boundary \( \bar{\alpha} \), the model returns very imprecise estimates of the four parameters \((\gamma_0, \gamma_1, \gamma_2, \gamma_3)\) of the exit-probability function. The point estimates of the parameters suggest that this imprecision is due to over-parametrization of the model. While the model permits the exit rule to be stochastic (i.e., the exit probability of a fund can lie between zero and one), the estimated function suggest that the mutual fund disappearance pattern in the model follows a far simpler rule. In the CAPM-based estimates, a fund always stays alive if the probability that the fund alpha is below \(-3.5\%\) per year is less than three fifths. However, as soon as the amount of probability mass below this critical threshold increases above three fifths, the exit probability jumps to one. (To be precise, the exit probability is zero at \( p_t = 0.597 \) but one at \( p_t = 0.601 \), where \( p_t \) is the amount of probability mass below the \(-3.5\%\) threshold.) Because a large set of parameter values \( \gamma_j \) can produce the same exit rule, the model returns imprecise estimates of these parameters.\(^{16}\) The step-function shape of the exit-probability function suggests that a

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\(^{16}\)I note that the exit rule also does not markedly shift over time in the CAPM-based estimates. Because the exit-probability function steps from zero to one, the shift of the intercept by \( \gamma_3 \) has only a small influence on the shape of the function. For example, by the tenth year, the location of the “step” in the exit probability has moved from 0.60 to just 0.62. By contrast, the exit rule does change over time in the three- and four-factor model-based estimates although the exact change is imprecisely estimated. These estimates indicate that as a fund matures, more probability mass below \( \bar{\alpha} \) is needed for it to disappear. The negative slope estimates in
simpler learning-based mechanism could replicate the fund disappearance rule. The generality of the model is not required to explain the mutual fund attrition patterns. A mutual fund disappears when the sum of evidence suggests that the fund’s alpha is too low relative to some critical threshold.

C. The Size of the Reverse Survivorship Bias

Panel C of Table IV reports on the means and percentiles of three different alpha distributions. I report on these distributions separately for the CAPM and the three- and four-factor models. The first column in each block, labeled “Data,” reports on the empirical distribution of alphas in the data. For example, the average alpha over all sample mutual funds, and with each alpha estimated using all available data, is $-1.11\%$ per year in the CAPM, $-1.55\%$ per year in the three-factor model, and $-1.54\%$ per year in the four-factor model. The second column reports on the observed alpha distributions that are computed by simulating from the model at the estimated parameter values. The means of the model-based (observed) alpha distributions are close to the data-based distribution. Moreover, the overall shapes of these distributions are very similar. The similarities in distributions are particularly striking in the CAPM-based estimates in which both the 1% and 99% percentiles are nearly the same between the data and the model. (I note that there are no moment conditions in the model to force the estimated model distribution to match these percentiles.)

Given that the observed distributions are very similar between the data and the model, it is now useful to use the model to examine how the true, unobserved alpha distribution differs from the observed distribution. What underlying true distribution of fund manager ability is responsible for the observed alpha distribution? The third column in each block, which reports on the true distribution, shows that the differences between the true and observed distributions are economically very significant. For example, starting from the CAPM-based estimates, the observed average alpha is $-1.17\%$ per year across but the true mean alpha is much higher, $-0.44\%$ per year. The difference between these numbers, 73 basis points per year, is the mean effect of the reverse survivorship bias. The estimated magnitude of this bias is similar when either the three- or four-factor model is used to estimate the input alphas.

Panel B for $\gamma_3$ are thus consistent with the increasing pattern in the critical alpha threshold in Figure 2.
The gap between the true and observed mean alpha is 104 basis points per year in the three-factor model and 88 basis points per year in the four-factor model. These estimates suggest that while the average mutual fund manager’s true alpha is negative, around $-0.5\%$ per year after fees, that manager’s alpha is not nearly as low as what the observed distribution of fund-specific alphas suggests.

The estimates of the true alpha distribution suggest that a minority of mutual managers may pick stocks well enough to cover the fees they impose on their investors. In the CAPM-based estimates, one percent of managers have alphas that are greater than 7.7% per year. However, adjustments for the size, book-to-market, and momentum factors result in a somewhat less optimistic assessment. In the three-factor model, the top one percent of managers have alphas greater than 2.4% per year and in the four-factor model, these right-tail alphas are greater than 2.3% per year. Thus, although the reverse survivorship bias significantly distorts the observed alpha distribution relative to the true alpha distribution, the model’s conclusion about fund managers’ abilities is not unlike those found in the literature: most mutual fund managers cannot pick stocks well enough to cover the costs they impose on their investors. However, a minority of managers appear to be able to do so.

The three- and four-factor model estimates of the size of the skilled-manager group lie between the estimates in Kosowski, Timmermann, Wermers, and White (2006) and Barras, Scaillet, and Wermers (2009). I note, however, that the benefit of the structural model approach is that it deals with the luck-versus-skill problem in an interesting way. Whereas the extant literature adjusts test statistics to account for the multiple-comparisons problem, a structural model can back out the true alpha distribution from a set of observables. The usual caveat, however, applies and the resultant skill estimates could be sensitive to the modeling assumptions. The structural model I consider in this section is fairly flexible, in particular with respect to the mutual fund attrition mechanism. Moreover, the model I consider matches not only the explicit moment conditions but also the general shape of the observed alpha distribution. These two considerations increase my confidence in the validity of the inferences drawn about the true alpha distribution.
D. The Speed of Learning about Mutual Fund Alphas

The estimated model suggests that some fund managers have enough skill to cover the costs they impose on their investors. An interesting application of the structural model is then to examine the search problem faced by a mutual fund investor. How easy, or how difficult, is it for an investor to find a positive-alpha fund? Two effects in the model complicate this task. First, the variance of risk-adjusted returns is very high, which means that investors in general resolve uncertainty only very slowly. Second, even if the true alpha is positive, the fund will disappear if it undergoes a sequence of negative idiosyncratic shocks. I assume, for the sake of this discussion, that if a fund disappears, then the investor has to restart the learning process from the beginning by picking another fund. However, I also consider the problem in a situation where the fund attrition mechanism has been turned off.

I run this experiment by taking the estimated model parameters and then simulate funds through the model. If a mutual fund investor can reject the null hypothesis $H_0 : \alpha < 0$ at 0.1, 0.05, or 0.01 significance levels, I record the time (in years) at which this rejection occurs. If a fund disappears, I record the amount of time the investor followed the fund and then give the investor another fund. I further control this experiment by always giving simulated investors funds from the positive tail of the alpha distribution. Specifically, each fund has a true alpha of at least 1% per year. (The simulated investors are not aware of this rule and believe that funds are drawn from the true alpha distribution.) Thus, I examine how long it would take for an investor to reject the null hypothesis that a fund has a non-positive alpha, given that the fund’s true alpha is at least 1% per year. In contrast to the difficulties faced in empirical studies, the simulated investors execute correctly sized tests about fund alphas. In reality both econometricians and mutual fund investors have to resolve the multiple-comparisons problem and correct for the fund return covariance structure to construct correctly sized test statistics to test hypotheses about fund alphas.

Panel D of Table IV reports on the time-to-rejection estimates separately for each of the three asset pricing models. I also report on results both with and without attrition. The without-attrition case makes an investor’s task easier by assuming that mutual funds never disappear and so investors never have to restart their learning processes. The CAPM-based
estimates show that if the fund attrition mechanism is turned off, it takes on average 8.7 years for an investor to reject the null hypothesis at 0.05 level. This number, however, blows up to 55.3 years when the attrition mechanism is turned back on. The reason for this increase is the high empirical attrition rate (i.e., one third of funds disappear in the first ten years), which means that some good funds disappear because of bad luck before they are identified as being good funds. The discovery of a fund with an alpha greater than 1% per year takes longer in both three- and four-factor models even when the attrition mechanism is turned off. The estimates for the attrition case are again very high, close to 60 years, indicating that it may take a long time for an investor discover a successful fund even in this idealized environment with constant alphas and (if need be) infinitely-lived fund managers.

I note that the case with fund attrition effectively assumes that an investor invests and follows only one fund at a time. In reality investors learn about all mutual funds simultaneously. If we continue to assume that investors are still given, unbeknownst to them, positive-alpha fund, then the time that it takes to discover a good fund with the attrition mechanism turned on lies between the no-attrition and attrition estimates of Panel D. As the depth of the cross section of mutual funds increases, and the distribution of true alphas remains unchanged, then the expected time it takes to reject the null decreases.

IV. Conclusions

The reverse survivorship bias affects the measurement of mutual fund managers’ stock-picking abilities. Because mutual funds often disappear following poor performance, some funds disappear because they experience negative idiosyncratic shocks and not because their true alpha is low. A fund that disappears because of a negative idiosyncratic shock leaves behind an alpha estimate that is too low. Because there is no offsetting mechanism that would eliminate just-lucky mutual funds, the observed distribution of alphas is biased downwards relative to the true distribution of alphas. In this paper I first solve a portfolio choice model for a representative fund investor that endogenizes mutual fund survival to theoretically demonstrate the economic significance of the reverse survivorship bias. I then estimate a variant of this model to measure the size of the bias in the CRSP mutual fund data. I find that the mean
effect of this bias is large, between 71 and 104 basis points per year depending on the asset pricing model. Thus, the average mutual fund manager’s alpha is not nearly as low as the observed alpha distribution suggests.

The question of whether any active fund managers have valuable information is one of the central questions in the empirical asset pricing literature. Although the average actively managed dollar must lose money because of the market-clearing constraint and transaction costs (French 2008), the necessity of these aggregate losses do not rule out the possibility that some active investors profit at the expense of their peers. A small number of mutual fund managers could, for example, be privy to a stream of signals that allows them to profit at the expense of other mutual funds, household investors, or pension funds. The reverse survivorship is of crucial importance in a study that examines whether any mutual managers trade on valuable information. Although the bias described in this paper does not influence the returns available to mutual fund investors, this average return also does not measure heterogeneity in mutual fund managers’ information. The correct answer (to the question of how many managers, if any, have enough skill to cover their costs) depends on the counterfactual of what would have happened had no fund actually disappeared following poor performance.

The ramifications of the reverse survivorship bias are not limited to the evaluation of mutual fund managers’ abilities. This bias also undoubtedly influences inferences about individual investors’ abilities. Similar to mutual funds, individual investors stop trading following poor performance. This sensitive to poor performance, which also could arise because individual investors learn about their abilities, must drive a wedge between investors’ observed returns and their true, unobserved abilities. The stylized survivorship-bias expression (2) shows that the size of the reverse survivorship bias is the product of three factors: This bias increases in the volatility of returns, the dispersion in survival times, and the correlation between returns and survival. This bias is thus probably large for individual investors because, first, individual investors hold under-diversified portfolios, which increases return volatility, and, second, because individual investors appear to be at least as sensitive to poor performance as mutual funds.

\[^{17}\text{See, for example, Seru, Shumway, and Stoffman (2009) and Linnainmaa (2009).}\]
REFERENCES


Figure 1. Cumulative Risk-Adjusted Returns for Mutual Funds Conditional on Fund Survival. I use time-series regressions to estimate the CAPM (Panel A) and Carhart’s four-factor model (Panel B) loadings for all mutual funds that either survive through year $t$ or that perish in year $t$. I then compute from these estimates the cumulative risk-adjusted returns up to the end of year $t$ or the month the fund disappears. The sample includes all 4,308 mutual funds that invest primarily in U.S. common stocks and that start on January 1984 or at any date after this. The sample covers a 25-year period up to the end of May 2009. Different share classes of the same fund are combined into a single fund. Thick solid lines denote average cumulative risk-adjusted returns of the funds that survive through the year; thin dashed lines denote average cumulative risk-adjusted returns of the funds that do not survive through the year. This figure plots cumulative risk-adjusted returns for even-numbered survival and disappearance years.
Figure 2. Critical Posterior Mean Boundary for Alpha (in Percentage Points per Year) in a Bayesian Portfolio Choice Model with Endogenous Fund Attrition. This figure shows the critical level for the posterior mean for alpha in a Bayesian portfolio choice model that causes a fund to shut down. In this model an infinitely lived representative mutual fund investor with log-utility can invest in a risk-free asset, the market portfolio, and an actively managed mutual fund. The investor is uncertain about the fund’s alpha but updates beliefs each month using Bayes’ rule. Each month the investor can abandon the existing mutual fund by paying a cost of $\kappa$. An investor then draws a new fund from the true alpha distribution and restarts the problem. The mean of the true alpha distribution ($\mu_\alpha$) is $-0.5\%$ and its standard deviation ($\sigma_\alpha$) is $0.682\%$; the expected return of the market portfolio ($\mu_m$) is $10\%$, and its standard deviation ($\sigma_m$) is $30\%$; the risk-free rate ($r_f$) is $5\%$; and the investor’s discount rate ($\beta$) is $0.95$. (These are annualized parameter values. Each period in the model is one month long.) The minimum mutual fund investment size as a fraction of wealth ($\bar{\theta}$) is $0.01$ and the variance of mutual fund returns is such that the variance of the posterior alpha distribution halves every fifth year. The solid line corresponds to a case in which the cost of closing a fund ($\kappa$) is $0.01$. The dashed line sets this cost to $0.0125$. 
Table I
Mutual Fund Alpha Estimates Conditional on Fund Survival, Percentage Points per Year

This table reports on the average alpha estimates for mutual funds based on whether the mutual fund survives or does not survive over $t$ years. The sample includes all 4,308 mutual funds that invest primarily in U.S. common stocks and that start on January 1984 or at any date after this. The sample covers a 25-year period up to the end of May 2009. Different share classes of the same fund are combined into a single fund. If a fund is still alive at the end of year $t - 1$, I estimate a time-series regression by using monthly returns up to the end of year $t$. If the fund disappears before the end of year $t$, I include returns up to the last available return. A fund that disappears by the end of year $t$ is assigned into a pool of year-$t$ dead funds; funds that are still alive at the end of the year are assigned into a pool of year-$t$ alive funds. This table reports on average $\alpha$'s for funds that either survive or do not survive year $t$. I use CAPM, three-factor model, and four-factor model to estimate alphas. $t$-values are reported in parentheses. The second to the last row reports the fraction of funds that survives at least $t$ years.

<table>
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<th>Model</th>
<th>Survive</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<th>Years after Inception</th>
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<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
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<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
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<td>-0.01</td>
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<td>0.38</td>
<td>0.32</td>
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<td>0.54</td>
<td>0.63</td>
<td>0.60</td>
<td>0.49</td>
<td>(1.81) (2.33) (3.21) (2.85) (4.43) (5.26) (6.40) (6.16) (5.22)</td>
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<tr>
<td></td>
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<td>-6.36</td>
<td>-4.92</td>
<td>-3.66</td>
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<td>-2.35</td>
<td>-2.43</td>
<td>-2.18</td>
<td>-1.98</td>
<td>-2.05</td>
<td>(-2.86) (-8.42) (-7.15) (-7.64) (-5.03) (-3.74) (-4.17) (-6.40) (-5.14) (-6.34)</td>
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<td>Yes</td>
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<td>0.33</td>
<td>0.33</td>
<td>0.20</td>
<td>-0.04</td>
<td>-0.07</td>
<td>-0.12</td>
<td>-0.12</td>
<td>-0.15</td>
<td>-0.28</td>
<td>(3.18) (2.36) (2.69) (1.76) (-0.35) (-0.68) (-1.33) (-1.38) (-1.65) (-3.04)</td>
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<td>-4.06</td>
<td>-3.74</td>
<td>-2.86</td>
<td>-2.39</td>
<td>-2.76</td>
<td>-2.66</td>
<td>-1.73</td>
<td>-1.68</td>
<td>(-2.90) (-6.27) (-6.71) (-9.06) (-5.63) (-4.06) (-5.64) (-9.00) (-5.61) (-5.31)</td>
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<td></td>
<td>Yes</td>
<td>0.95</td>
<td>0.31</td>
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<td>-0.15</td>
<td>-0.15</td>
<td>-0.23</td>
<td>-0.21</td>
<td>-0.21</td>
<td>-0.35</td>
<td>(5.03) (2.29) (2.03) (0.97) (-1.55) (-1.70) (-2.75) (-2.53) (-2.38) (-4.08)</td>
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<td>Four-Factor Model</td>
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<td>-4.02</td>
<td>-3.91</td>
<td>-3.51</td>
<td>-3.22</td>
<td>-2.61</td>
<td>-2.72</td>
<td>-2.85</td>
<td>-1.87</td>
<td>-1.98</td>
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<td>Survival Rate</td>
<td></td>
<td>0.989</td>
<td>0.960</td>
<td>0.922</td>
<td>0.889</td>
<td>0.846</td>
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<td>0.735</td>
<td>0.698</td>
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<td>N</td>
<td>4,269</td>
<td>3,954</td>
<td>3,631</td>
<td>3,215</td>
<td>2,923</td>
<td>2,591</td>
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<td>2,074</td>
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<td>1,569</td>
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Table II
Mutual Fund Survival and Alphas: Probit Regressions

This table reports on Fama-MacBeth style probit regressions of mutual fund disappearance on rolling estimates of fund alphas. The sample includes all 4,308 mutual funds that invest primarily in U.S. common stocks and that start on January 1984 or at any date after this. The sample covers a 25-year period up to the end of May 2009. Different share classes of the same fund are combined into a single fund. The dependent variable in this table’s exit regressions takes the value of one if a fund disappears during year $t$ and zero if it does not. I estimate the exit regression separately for each month (based on how long each fund has existed) and drop out months in which no mutual fund disappears. For example, one regression uses data on funds that have existed for eight years and two months. This table reports on the averages and $t$-values of the resultant 127 cross-sectional estimates. I estimate fund alphas from monthly time-series regressions by using all data from the first month of fund returns up to the last available return (Panel A). Panel B reports on estimates that lag the alpha estimates by a year. The underlying panel data contain 326,815 observations.

<table>
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<th>Three-Factor Model</th>
<th>Four-Factor Model</th>
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<td>Intercept</td>
<td>−2.71</td>
<td>−2.73</td>
<td>−2.72</td>
</tr>
<tr>
<td></td>
<td>(−169.40)</td>
<td>(−120.91)</td>
<td>(−179.86)</td>
</tr>
<tr>
<td>Slope Coefficient</td>
<td>−43.88</td>
<td>−46.26</td>
<td>−47.39</td>
</tr>
<tr>
<td></td>
<td>(−12.20)</td>
<td>(−6.88)</td>
<td>(−11.42)</td>
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<table>
<thead>
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<th></th>
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<th>Three-Factor Model</th>
<th>Four-Factor Model</th>
</tr>
</thead>
<tbody>
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<td>Intercept</td>
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<td>−2.70</td>
<td>−2.68</td>
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<tr>
<td></td>
<td>(−178.15)</td>
<td>(−97.70)</td>
<td>(−182.63)</td>
</tr>
<tr>
<td>Slope Coefficient</td>
<td>−38.98</td>
<td>−41.93</td>
<td>−41.97</td>
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<tr>
<td></td>
<td>(−11.12)</td>
<td>(−5.09)</td>
<td>(−9.47)</td>
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</table>
Table III
Calibration Results for a Bayesian Portfolio Choice Model with Endogenous Fund Attrition

This table reports on calibration results for a Bayesian portfolio choice model. In this model an infinitely lived representative mutual fund investor with log-utility can invest in a risk-free asset, the market portfolio, and an actively managed mutual fund. The investor is uncertain about the fund’s alpha but updates beliefs each month using Bayes’ rule. Each month the investor can abandon the existing mutual fund by paying a cost of $\kappa$. An investor then draws a new fund from the true alpha distribution and restarts the problem. The mean of the true alpha distribution ($\mu_\alpha$) is $-0.5\%$ and its standard deviation ($\sigma_\alpha$) is $0.682\%$; the expected return of the market portfolio ($\mu_m$) is $10\%$, and its standard deviation ($\sigma_m$) is $30\%$; the risk-free rate ($r_f$) is $5\%$; and the investor’s discount rate ($\beta$) is $0.95$. (These are annualized parameter values. Each period in the model is one month long.) Panel A reports on ten-year survival rates based on alternative choices for the remaining three fund-specific parameters. Both the minimum mutual fund investment size ($\bar{\theta}$) and the cost of switching a fund ($\kappa$) take values $0$, $0.01$, and $0.02$. The variance of mutual fund returns is such that the variance of the posterior distribution halves every second, fifth, or tenth year. Panel B reports on average alphas conditional on fund survival. The first specification in this panel sets $\bar{\theta} = 0.01$, $\kappa = 0.01$, and the speed of learning to five years. The second specification increases the cost of switching to $\kappa = 0.0125$. Panel C simulates from the model, using the same two specifications as in Panel B, and reports on the actual alpha distribution and the observed distribution. The observed distribution is different from the true distribution because of the endogenous disappearance of poorly performing funds. The results in this table are based on simulating 100,000 funds through the model. I simulate at most 25 years of monthly returns for each fund.

\begin{table}[h]
\centering
\begin{tabular}{l|ccc|ccc|ccc}
\hline
 & \multicolumn{6}{c}{Minimum Investment Size $\theta$} \\
 & \multicolumn{3}{c}{0.00} & \multicolumn{3}{c}{0.01} & \multicolumn{3}{c}{0.02} \\
\hline
\multirow{3}{*}{Speed of Learning} & \multicolumn{3}{c}{Strike Price $\kappa$} & \multicolumn{3}{c}{Strike Price $\kappa$} & \multicolumn{3}{c}{Strike Price $\kappa$} \\
\cline{2-10}
 & 0.00 & 0.01 & 0.02 & 0.00 & 0.01 & 0.02 & 0.00 & 0.01 & 0.02 \\
2 years & 3.40 & 29.60 & 47.39 & 3.56 & 29.32 & 47.59 & 3.51 & 29.68 & 47.52 \\
5 years & 4.37 & 58.87 & 83.87 & 4.46 & 58.76 & 83.52 & 4.28 & 84.43 & 97.85 \\
10 years & 5.16 & 96.37 & 99.97 & 5.45 & 96.21 & 99.98 & 5.26 & 96.11 & 99.98 \\
\hline
\end{tabular}
\caption{Ten-Year Mutual Fund Survival Rates (\%)}
\end{table}
Panel B: Average Alphas Conditional on Fund Survival, Percentage Points per Year

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>Survive</th>
<th>Years after Inception</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>1</td>
</tr>
<tr>
<td>$\theta = 0.01$</td>
<td>Yes</td>
<td>-0.41</td>
</tr>
<tr>
<td>$\kappa = 0.01$</td>
<td>Speed = 5 yrs</td>
<td>-18.97</td>
</tr>
<tr>
<td>$\bar{\theta} = 0.01$</td>
<td>Yes</td>
<td>-0.46</td>
</tr>
<tr>
<td>$\kappa = 0.0125$</td>
<td>Speed = 5 yrs</td>
<td>-21.60</td>
</tr>
</tbody>
</table>

Panel C: Observed versus True Alpha Distributions

<table>
<thead>
<tr>
<th></th>
<th>$\theta = 0.01$</th>
<th>$\theta = 0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\kappa = 0.01$</td>
<td>$\kappa = 0.0125$</td>
</tr>
<tr>
<td>Speed = 5 yrs</td>
<td>Obs. True</td>
<td>Obs. True</td>
</tr>
<tr>
<td>Mean</td>
<td>-1.601 -0.502</td>
<td>-1.310 -0.497</td>
</tr>
<tr>
<td>Percentiles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>-14.257 -5.976</td>
<td>-12.314 -5.993</td>
</tr>
<tr>
<td>5%</td>
<td>-8.904 -4.395</td>
<td>-7.889 -4.365</td>
</tr>
<tr>
<td>25%</td>
<td>-3.604 -2.096</td>
<td>-3.276 -2.089</td>
</tr>
<tr>
<td>50%</td>
<td>-1.150 -0.505</td>
<td>-1.010 -0.513</td>
</tr>
<tr>
<td>75%</td>
<td>1.181 1.081</td>
<td>1.242 1.095</td>
</tr>
<tr>
<td>95%</td>
<td>3.736 3.377</td>
<td>3.788 3.416</td>
</tr>
<tr>
<td>99%</td>
<td>5.503 5.019</td>
<td>5.516 4.970</td>
</tr>
</tbody>
</table>
Table IV
Estimates of the Structural Learning Model

This table reports on the estimates of the structural learning-based model of mutual funds. The model is estimated using data on all 4,308 mutual funds that invest primarily in U.S. common stocks and that start on January 1984 or at any later date. Each mutual fund’s alpha is drawn from a normal distribution with a mean of $\mu$ and a variance of $\sigma^2$. Each mutual fund generates monthly return observations $\tilde{R}_{i,t} = \alpha_i + \tilde{\epsilon}_{i,t}$, where $\tilde{\epsilon}_{i,t}$ is normally distributed with a mean of zero and a variance of $\sigma^2_e$. The market uses each risk-adjusted monthly return realization to update from the prior distribution to a posterior distribution. The probability that the fund disappears after month $t$ is a function of the posterior distribution $N(m_{i,t}, v_t)$. Investors first compute the amount of mass below some critical level of alpha $\bar{\alpha}$. A fund disappears after period $t$ with probability $\xi(p_t, t)$, where $\xi(\cdot)$ is approximated by $\xi(p_t, t) \approx \tilde{\xi}(p_t, t) = \max(\min(\gamma_0 + \gamma_1 p_t + \gamma_2 p_t^2 + \gamma_3 t, 1), 0)$. The model is estimated using simulated method of moments. The first five moment conditions represent the means of the alpha distributions for the funds that survive through years two, four, six, eight, and ten. The next five moment conditions represent the means of the alpha distributions for funds that perish in the same two-year periods. The eleventh moment condition is the ten-year survival rate. The last two moment conditions are the mean and the variance of the overall alpha distribution. I estimate alphas using at most ten years of data. All parameters are annualized. The mean-alpha moment conditions are multiplied by 100 and the variance moment condition is multiplied by 10,000 for the ease of reporting. I use the optimal weighing matrix to estimate the model. I estimate the model three times by using the CAPM, the three-factor model, and the four-factor model. Panel A compares simulated moments to sample moments, Panel B reports on the (annualized) structural parameter estimates, Panel C tabulates three alpha distributions for each asset pricing model (the actual distribution in the data, the observed distribution in the model, and the true (unobserved) distribution), and Panel D reports on the average time (in years) it takes for an investor to reject the null hypothesis that a fund’s alpha is negative given that the true alpha is greater than 1% per year. Columns marked ‘Attrition’ in Panel D correspond to the estimated model with fund attrition; columns marked ‘No Attrition’ shut down the attrition mechanism.
Panel A: Actual Moments versus Simulated Moments

<table>
<thead>
<tr>
<th>Moment Condition</th>
<th>CAPM</th>
<th>Three-Factor</th>
<th>Four-Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data  Model</td>
<td>Data  Model</td>
<td>Data  Model</td>
</tr>
<tr>
<td>Alpha Estimates for Disappearing</td>
<td>-0.488 -0.363</td>
<td>-0.431 -0.366</td>
<td>-0.304 -0.328</td>
</tr>
<tr>
<td>Funds</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Years 1–2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Years 3–4</td>
<td>-0.364 -0.351</td>
<td>-0.323 -0.309</td>
<td>-0.312 -0.312</td>
</tr>
<tr>
<td>Years 5–6</td>
<td>-0.222 -0.268</td>
<td>-0.229 -0.243</td>
<td>-0.248 -0.250</td>
</tr>
<tr>
<td>Years 7–8</td>
<td>-0.190 -0.198</td>
<td>-0.221 -0.200</td>
<td>-0.226 -0.210</td>
</tr>
<tr>
<td>Years 9–10</td>
<td>-0.175 -0.154</td>
<td>-0.156 -0.165</td>
<td>-0.174 -0.178</td>
</tr>
<tr>
<td>Alpha Estimates for Surviving</td>
<td>0.000 0.009</td>
<td>0.029 0.005</td>
<td>0.027 -0.004</td>
</tr>
<tr>
<td>Funds</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Years 1–2</td>
<td>0.034 0.033</td>
<td>0.018 0.001</td>
<td>0.008 -0.006</td>
</tr>
<tr>
<td>Years 3–4</td>
<td>0.039 0.042</td>
<td>-0.006 -0.002</td>
<td>-0.012 -0.009</td>
</tr>
<tr>
<td>Years 5–6</td>
<td>0.054 0.047</td>
<td>-0.009 -0.004</td>
<td>-0.017 -0.011</td>
</tr>
<tr>
<td>Years 7–8</td>
<td>0.043 0.050</td>
<td>-0.022 -0.006</td>
<td>-0.027 -0.013</td>
</tr>
<tr>
<td>Ten-Year Survival Rate</td>
<td>0.666 0.671</td>
<td>0.666 0.669</td>
<td>0.666 0.667</td>
</tr>
<tr>
<td>Mean of the Observed Alpha</td>
<td>-0.082 -0.090</td>
<td>-0.117 -0.121</td>
<td>-0.115 -0.116</td>
</tr>
<tr>
<td>Distribution</td>
<td>0.250 0.246</td>
<td>0.246 0.250</td>
<td>0.237 0.242</td>
</tr>
<tr>
<td>Variance of the Observed Alpha</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Distribution</td>
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</table>
Panel B: Structural Parameter Estimates

<table>
<thead>
<tr>
<th></th>
<th>CAPM</th>
<th>Three-Factor</th>
<th>Four-Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EST</td>
<td>SE</td>
<td>EST</td>
</tr>
<tr>
<td>( \mu (%) )</td>
<td>-0.443</td>
<td>(0.155)</td>
<td>-0.578</td>
</tr>
<tr>
<td>( \sigma (%) )</td>
<td>1.015</td>
<td>(0.275)</td>
<td>0.366</td>
</tr>
<tr>
<td>( \sigma_x (%) )</td>
<td>3.860</td>
<td>(3.642)</td>
<td>3.067</td>
</tr>
<tr>
<td>( \bar{\alpha} (%) )</td>
<td>-3.518</td>
<td>(0.590)</td>
<td>-2.794</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>1.118</td>
<td>(0.967)</td>
<td>21.514</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>-36.490</td>
<td>(172.194)</td>
<td>-137.528</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>6.076</td>
<td>(27.540)</td>
<td>143.903</td>
</tr>
<tr>
<td>( \gamma_3 )</td>
<td>-0.478</td>
<td>(0.496)</td>
<td>-29.714</td>
</tr>
<tr>
<td>( J )-test, ( \chi^2 ) (p-value)</td>
<td>10.964</td>
<td>(0.052)</td>
<td>15.268</td>
</tr>
</tbody>
</table>

Panel C: Observed versus True Alpha Distributions

<table>
<thead>
<tr>
<th></th>
<th>CAPM</th>
<th>Three-Factor</th>
<th>Four-Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Obs.</td>
<td>True</td>
</tr>
<tr>
<td>Mean</td>
<td>-1.107</td>
<td>-1.173</td>
<td>-0.443</td>
</tr>
<tr>
<td>Percentiles</td>
<td></td>
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<tr>
<td>50%</td>
<td>-0.754</td>
<td>-0.537</td>
<td>-0.443</td>
</tr>
<tr>
<td>75%</td>
<td>1.365</td>
<td>2.051</td>
<td>1.929</td>
</tr>
<tr>
<td>95%</td>
<td>6.457</td>
<td>8.144</td>
<td>5.341</td>
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</table>
Panel D: Average Rejection Times (in Years) for Null Hypothesis $H_0 : \alpha < 0$

<table>
<thead>
<tr>
<th>Significance Level</th>
<th>CAPM</th>
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<th>Four-Factor</th>
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<tbody>
<tr>
<td></td>
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</tr>
<tr>
<td>0.10</td>
<td>54.02</td>
<td>57.61</td>
<td>57.54</td>
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<tr>
<td>0.05</td>
<td>55.29</td>
<td>59.62</td>
<td>59.49</td>
</tr>
<tr>
<td>0.01</td>
<td>57.73</td>
<td>63.63</td>
<td>63.42</td>
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</tbody>
</table>