Knightian Uncertainty and Interbank Lending

Matthew Pritsker∗

September 20, 2010

Abstract

The bursting of the housing price bubble during 2007 and 2008 was accompanied by high interbank spreads, and a partial breakdown of interbank lending. This paper models Knightian uncertainty over banks risk exposures and shows how it may have contributed to the collapse of interbank lending. The main finding is that institutional aspects of the Fed Funds market in the U.S. help make it robust to Knightian uncertainty; however, the market may sometimes collapse — and private incentives to restart it may be insufficient. In some circumstances government inspection of banks accompanied by the release of information about risk exposures can restart markets and improve welfare by internalizing an externality associated with reducing economy-wide uncertainty during a crisis. The results also show that collapses due to uncertainty are less likely ex-ante and less costly to fix ex-post when there is better publicly available information on aggregate exposures of core banks within the financial system. The paper proposes ex-ante and ex-post transparency initiatives to improve market function. The success of “transparency initiatives” in restarting markets depends on the financial architecture of bank linkages. This suggests that public policy aimed at resuscitating markets ex-post should focus on both the initiatives and the optimal design of the ex-ante financial architecture.

Keywords: Banking, Risk, Knightian Uncertainty

JEL Classification Numbers: G21, G32

∗Board of governors of the Federal Reserve System. The views expressed in this paper are those of the author but not necessarily those of the Board of Governors of the Federal Reserve System or other members of its staff. This work was begun while the author was visiting NYU Stern School of Business. The author thanks without implicating Thomas Sargent, Anthony Saunders, Viral Acharya, Tanju Yorulmazer, Douglas Gale, Rangarajan Sundaram, Karan Bhanot, Wenying Jiangli, Andrei Kirilenko, Tobias Adrian, Borys Grochulski, and seminar participants at the Federal Reserve Bank of New York, the Federal Reserve Board, the Federal Reserve Bank of Kansas City, the Federal Reserve Bank of Atlanta, and the Bank of Canada for useful conversations and comments. Contact info: The Federal Reserve Board, Mail Stop 91, Washington DC 20551. Phone: (202) 452-3534. Email: mpritsker@frb.gov.
1 Introduction

The U.S. housing price boom of 2000-2006 was accompanied by the proliferation of housing backed collateral in the form of mortgage-debt that was repackaged, and resold to investors and banks, both in the US, and around the world. In the wake of the collapse of housing prices, a global financial crisis ensued in which major institutions failed, financial market volatility rose to historically unprecedented levels, interbank spreads were sharply elevated, interbank credit extension was sharply reduced, and governments needed to take many actions to address the crisis.  

In this paper I argue that an important aspect of the crisis beyond the increase in risk, was severely heightened uncertainty that resulted from the interaction of a riskier economic environment and a build-up of structural economic uncertainty, which means incomplete knowledge of the structure of the economy. In the present crisis this was exemplified by incomplete knowledge of the risk exposures of financial institutions and instruments to various sources of risk in general, and housing risk in particular. Intuitively, precise knowledge of risk exposures is not important when risk is low, but can become important when risk is elevated, and lead to more cautious behavior that further exacerbates a crisis. In the 2007-2009 crisis, when housing prices declined and housing risk became elevated, it was unclear who had taken large losses and who was still exposed. The uncertainty made counterparty risk evaluation problematic, contributed to market dysfunction, and exacerbated the crisis.

To help address this and potential future crisis, this paper proposes policy steps that reduce uncertainty. Because uncertainty reduction is a public good, these actions will be underprovided by the private sector both ex-post, i.e. during a crisis, as well as ex-ante. The paper shows that during a crisis the government may be needed to step-in and reduce uncertainty. A variant of bank stress tests, such as those used by the US in the crisis of 2007-2009, is recommended. In addition, we show that uncertainty reduction before a crisis is also beneficial. We present a proposal for the release of information on risk limits and the aggregated across banks risk exposures of systemically important banks, known as core banks hereafter. This proposal has the advantage that it keeps the risk exposures of individual banks private, but by revealing information on risk limits and risk aggregates helps market participants place bounds on the financial health of the average systemically important bank. In the context of stylized examples, we show these measures reduce the likelihood of financial crises, and reduce the costs of restarting markets if a financial crisis should occur. These points are illustrated in the context of the interbank market, but apply more generally.

If structural uncertainty can impede the function of markets, then market institutions should appear that reduce the effects of structural uncertainty. A second contribution of this paper is that it theoretically studies the organization of the interbank market from a structural uncertainty perspective and on the basis of structural uncertainty alone provides

\footnote{For details on money market spreads in the U.S., Europe, and Japan, see Heider et al. (2009), and Taylor and Williams (2009).}
an argument for why the interbank market in the US has a multi-tiered structure in which many large banks trade with each other anonymously in the top tier of the market, while small banks are largely excluded from the top tier as borrowers, but can occasionally borrow from large banks on a bilateral basis. The analysis also shows why the interbank market is typically very resilient to shocks to small sectors of the economy, but not to large sectors.

Before proceeding further it is useful to clarify the distinction between risk, uncertainty, and structural uncertainty as they are used here. A decision maker faces risk if the outcomes from his decisions are random. He faces uncertainty if the outcomes are random and he does not know the probabilities of the outcomes. Uncertainty, as it is used here is also referred to as Knightian uncertainty. An important source of uncertainty is incomplete knowledge of the structure the economic environment, which is referred to above as structural uncertainty.

Uncertainty can be harmful because experiments such as those of Ellsberg (1961) show that when agents are confronted with gambles where they do not know probability distributions, they tend to behave more cautiously. In market contexts, this cautious behavior can impede market function. A canonical example illustrates how uncertainty arises naturally in banking contexts, how it is tied to risk and structural uncertainty, and why it can be harmful.

Consider a loan officer who can make a loan in one of two communities, A or B. In community A there are two types of borrowers, H and L. Both types are indistinguishable but H-types are high-risk and default with probability $p_H$ and L-types are low risk and default with probability $p_L$, where $p_H > p_L$. In community A the loan officer knows the proportions of H and L types are $\pi_H(A)$ and $1 - \pi_H(A)$. If the loan officer lends in community A, the probability that his borrower is an H type is just its proportion in the population, $\pi_H(A)$. Community B is just like community A, except that the loan officer has limited information on the proportion of H and L-type borrowers that takes the form $\pi_H(B) \in [\pi_H(A), \pi_H(B)]$. This means he has a range for the proportion of high and low risk borrowers, but does not have beliefs about the relative likelihood of elements in the range that are sufficiently well formed that they can be described by a probability distribution. This lack of well formed beliefs about $\pi_H(B)$ is a source of structural uncertainty in community B.

The loan officer’s knowledge implies that lending in community A only involves risk since the loan officer knows the probability a loan in A will default is given by

$$PD(A) = \pi^H(A)p^H + [1 - \pi^H(A)]p^L. \tag{1}$$

Conversely, in community B, because the loan officer does not know or have a probability distribution for $\pi^H(B)$, he does not know $PD(B)$. Nevertheless, using equation 1, for every possible $\pi^H(B) \in [\pi^H(A), \pi^H(B)]$ he can calculate a probability of default. Therefore, he believes there is a range of plausible values for $PD(B)$ given by:

$$PD(B) \in [\pi^H(B)p^H + (1 - \pi^H(B))p^L, \pi^H(B)p^H + (1 - \pi^H(B))p^L] \tag{2} \in [PD(B), PD(B)].$$

This means, if the loan officer was asked to assess the probability a loan will default in community B, he might state he does not know, but believes it could range from 1 to 3 percent. The fact he cites a range is the consequence of his structural uncertainty and how
it interacts with the the risk of the two borrower types. Even though the loan officer does not know $PD(B)$, if the upper bound of his interval for it is low enough, and the spread he can charge in community $B$ is high enough, he may rationally decide to extend the loan.

If the loan officer sets his spread based on the high end of his range for $PD(B)$, he is choosing it in an uncertainty averse fashion, which guarantees his loan spread will cover the highest possible default probability. This may not be too harmful to borrowers in community $B$ if in most circumstances the difference in risk between high and low risk borrowers is small, but it can become problematic during times when the differences in risk between the borrower types become high since during such times low risk borrowers are forced to pay high spreads or may choose not to borrow at all.

Gilboa and Schmeidler (1989) provide an axiomatic foundation for uncertainty averse behavior when agents uncertainty about probabilities is modeled by assuming that agents hold multiple prior distributions over economic state-variables, such as the proportion of high and low risk borrowers, and then make choices based on worst case beliefs about those priors.\(^2\) Much of the analysis in this paper is an application of their framework within a very simple class of multiple priors.\(^3\)

The same problems that uncertainty and uncertainty averse behavior can cause for borrowers in community $B$ can also occur in the interbank market because a creditor bank’s may have incomplete knowledge about a borrowing bank’s portfolio weights and therefore like the loan officer be uncertain about the bank’s probability of default.

When one bank extends a loan to another, the level of uncertainty is endogenous since creditor banks could seek more information before lending or borrowing banks could provide it. Many interbank transactions involve substantial uncertainty, especially in the core of the Fed Funds market, where the largest banks have hundreds of billions to trillions of dollars of assets, and often extend unsecured loans to each other on very short notice (minutes to hours) through an anonymous brokered market that leaves little time for lending banks to learn about borrowing banks risk exposures.

This paper shows that the institutional arrangements in the Fed Funds market make it possible for the market to support lending when there is a significant amount of uncertainty. It also provides an explanation for why in the recent financial crisis interbank market lending, especially for longer term loans, dried up, and the spreads on interbank loans climbed to unprecedented levels.

When markets collapse because of uncertainty about risk exposures, banks have incentives to reduce uncertainty about their own conditions, but in some states of nature the private

\(^2\)In Gilboa and Schmeidler, GS, the uncertainty aversion is extreme, but it is the most simple method for modeling it. Klibanoff et. al. (2005) provide an alternative smooth method for modeling uncertainty aversion which is less extreme than GS’s approach, but nests it as a special case

\(^3\)The example can also be understood as an application of multiple priors in which each of the multiple priors assigns probability one to a single value of $\pi^H(B)$ and 0 to the other values. The ideas in the paper are illustrated in a static setting for simplicity. For treatments of Knightian Uncertainty in a dynamic setting see Hansen and Sargent (2008), Chen and Epstein (2002), and Epstein and Schneider (2003).
costs of reducing uncertainty exceed the private benefits—and as a result the markets will not restart on their own. In some circumstances a government policy that sequentially audits individual banks and releases information about their health and their positions produces positive externalities that can reduce the economy wide level of uncertainty, restarts markets, and raises welfare. In addition to these ex-post steps, measures that improve the quality of information on aggregate exposures among core banks can help prevent collapses ex ante. The efficacy of such policies depends on the architecture of the financial system—who is connected to whom, and who knows about it. Therefore, the broader financial architecture is important for formulating policies to mitigate the effects of structural economic uncertainty.

The paper is related to literature on Knightian uncertainty, the financial crisis of 2007-2009, and to the literature on interbank markets.\footnote{The paper is also related to the literature on corporate transparency and asset pricing. Duffie and Lando (2001) theoretically show and Yu (2005) empirically literature confirms that imprecise accounting figures can affect firms borrowing costs, especially for short-rated debt.}

Because Knightian uncertainty and uncertainty averse behavior can be damaging to markets, government and market institutions have emerged to reduce that uncertainty. Easley and O’Hara (2009) show that deposit insurance can be understood a response to uncertainty averse behavior by small depositors that curtails the behavior not by eliminating the source of the uncertainty, but by guaranteeing that small depositors will be repaid even if the bank fails.\footnote{In a different setting Caballero and Krishnamurthy (2008) model government policy during a stylized crisis when there is uncertainty over the timing of liquidity shocks.} This paper shows that other institutions have emerged to reduce uncertainty among large banks, most notably the tiered structure of the interbank market, as well as government actions such as the recent stress capital exercise in the United States, or the Bank Holiday of 1933. In complementary work Caballero and Simsek (2009) focus on market collapses due to uncertainty over the shape of the network of interbank exposures. Policies to move towards central clearing of OTC derivatives can be interpreted as a government action to reduce uncertainty over interbank exposures.

The most closely related research on the crisis is a series of papers by Gary Gorton and coauthors [Gorton (2008), Gorton (2009), Metrick and Gorton (2009), and Dang, Gorton, and Holmstrom (2009)]. An important theme in the Gorton et. al. papers is that the value of AAA tranches that were used as collateral in interbank transactions became informationally sensitive to the portfolio composition of the assets backing the tranches when the value of some of those assets declined.\footnote{Gorton and his coauthors terminology defines the sensitivity of asset value to a state variable as the informational sensitivity to the variable.} The heightened information sensitivity created incentives for market participants to gather information; which created asymmetric information that causes repo spreads and haircuts to widen, and the markets for AAA tranches to collapse.

This paper departs from the Gorton papers in three ways. First, their approach is based on increased informational sensitivity and asymmetric information while mine is based on increased informational sensitivity and uncertainty averse behavior. In my story markets collapsed because risk increased, many participants did not understand (i.e. were uncertain...
about) the economic environment, and based on their worst fears required large spreads and haircuts in order to take positions. In their story, large spreads were required because market participants were worried that someone understood the environment better than they did. The approaches are not mutually exclusive. Future empirical analysis is needed to distinguish between them.

The second difference concerns the setting, and implications for economic policy. In an endowment economy, Dang, Gorton, and Holmstrom (2009) show that symmetric ignorance about the value of endowments is better than symmetric information. A consequence of their framework is that more transparency about banks is not desirable. In the present framework, the answer is very different because I consider a production economy in which some production cannot occur unless information about the banks is of sufficient quality for financial intermediation to take place. A natural consequence is that better information about banks restores their ability to lend and improves welfare.

A final distinction is that the Gorton papers focus on the collapse of secured (repo) borrowing, while I focus on the collapse of unsecured arrangements and on how to resuscitate and improve the unsecured markets.

There is a large literature on interbank markets. Closely related research models models interbank markets as having collapsed during the crisis due to adverse selection and asymmetric information [Heider et. al (2009)]. Additionally, small banks limited participation as borrowers in unsecured interbank markets and their participation in secured markets is interpreted as implying they face adverse selection problems when borrowing [Allen, Persi-tani, and Saunders (1989)]. This paper questions this interpretation because large banks are often much more opaque and complicated than small banks, but large banks often borrow in unsecured interbank markets. Instead, I argue that small banks are excluded and some large banks are included because both steps can reduce the effects of Knightian uncertainty on spreads in the top-tier of the interbank market. An additional contribution of this paper is that to the best of my knowledge it is one of the first to study why the interbank market has an anonymous brokered tier and a tier that involves bilateral trading.

The remainder of the paper consists of 5 sections. Section 2 illustrates the key ideas of the paper in the context of a canonical example. Section 3 presents our model of the economy and studies interbank loan spreads and market breakdown in the context of stripped down model of the interbank market. Section 4 presents our general model and uses it to analyze steps the government can take to fix markets that breakdown, and institutional features of the Federal Funds market that help to prevent breakdown. It also discusses government policy when agents are uncertainty averse versus when they are expected utility maximizers with well formed beliefs. Section 5 discusses the role of financial architecture in government efforts to prevent breakdown. A final section concludes.

The main focus of Dang, Gorton, and Holmstrom (2009) is the optimality of debt as a vehicle for raising financing while avoiding adverse selection.
2 A Canonical Example

This section completes the canonical example from the introduction. An added twist is that the default probabilities of the high and low risk loans, $p^H$ and $p^L$ depend on macro-economic conditions, $F$ that are known at the time a loan is extended as follows:

$$p^H = \alpha^H + \beta^H F,$$
$$p^L = \alpha^L + \beta^L F,$$

with $\alpha^H > \alpha^L > 0$ and $\beta^H > \beta^L > 0$.

$F$ takes the value 0, when times are good, and is equal to the random variable $f > 0$ when conditions are poor, such as during a recession.

If loan officers in community $B$ compete for business but are also risk neutral and uncertainty averse, then they will set loan spreads $S(B)$ based on the high end of the range for the probability of default, given in equation 2: $S(B) = PD(B)\cdot LGD(B)$, where $LGD(B)$ is the loss given default on loans in community $B$. Using this spread, the loan officers expected return on the loan will be the same as the risk free rate when expectations are taken using their worst beliefs about default probabilities in community $B$.\(^8\) Letting $PD(B)$ be the true probability of default in community $B$, the spread can be decomposed into a default premium and into an additional positive premium for uncertainty:\(^9\):

$$S(B) = PD(B)\cdot LGD(B)$$
$$= PD(B)\cdot LGD(B) + [PD(B) - PD(B)]\cdot LGD(B)$$
$$= \text{Default Premium} + \text{Uncertainty Premium}.$$

The uncertainty premium can be further decomposed as the product of an uncertainty component and a risk component:

$$\text{Uncertainty Premium} = [\pi^H(B) - \pi^H(B)] \times [(p^H - p^L) \times LGD(B)]$$
$$= [\text{Uncertainty Component}] \times [\text{Risk Component}],$$

The decomposition shows structural uncertainty and risk interact in determining loan spreads. When risk is low ($F = 0$) if $\alpha^H$ is not much different from $\alpha^L$, then uncertainty premia can be low and borrowers in community $B$ may not care about it. While during bad times ($f > 0$) uncertainty premia can be very high, leading to a borrowing collapse.

\(^8\)With no structural uncertainty, there would be a unique prior over the distribution of $\pi^H(B)$, and the spread would be based on the expected value of $\pi^H(B)$, which would produce a spread less than $PD(B)$.

\(^9\)The default premium is for expected losses with $PD(B)$ known. The spread does not contain a risk premium because loan officers are risk neutral.
When borrowing collapses, the provision of information on $\pi^H(B)$ may help it restart. But, private incentives to provide the information may prove insufficient. Thus, in some cases government action may be needed to reduce the uncertainty and resuscitate lending. The analysis that follows expands on these ideas, but applies them in the context of the interbank market.

3 Model

Our basic framework is of a stylized competitive economy that has $M$ economic sectors, and $2N$ large banks that make loans to those sectors for the purposes of long term investments and for short-term liquidity needs. There are three dates, 0, 1, and 2. At date 0 the banks raise funds from depositors and equity holders and invest those funds in long-term loans that mature at date 2. At date 1, news arrives about the performance of the macro-economy. In addition each bank experiences either a funding shock that provides it with more deposits, or a lending shock that provides it with opportunities to make short-term loans, but not both. Following the news and shocks, banks use the interbank market to channel funds from those banks that have excess funds to those banks that have excess lending opportunities. The heart of the paper analyzes the institutions in the Federal Funds market that facilitate this transfer of funds at date 1 in the presence of uncertainty. To close the model, at date 2, the returns on banks loan portfolios are realized, and banks pay back their stakeholders if fully solvent—or default and make partial payments if not solvent.

3.1 Date 0

At date 0, each large bank $i = 1, \ldots, 2N$ is endowed with a fixed amount of equity funding $E_i$, and then chooses to raise deposit funding $D_i$, in order to fund a long-term asset portfolio of size $A_i$ ($A_i = D_i + E_i$). Including deposit insurance premium payments, for each dollar of deposits the bank pays interest $R^D$ that we assume is fixed and does not depend on the banks portfolio weights.$^{10}$

The long-term asset portfolio consists of investments in $M$ sector-portfolios. Bank $i$’s portfolio weight is denoted by the $M$-vector $\omega_i$. Each sector portfolio consists of loans to a positive measure of small borrowers in each of the $M$ sectors of the economy. The gross return per dollar received at date 2 for $\$1$ invested in the $m$’th sector portfolio at date 0 is $R_m$. The return on the vector of sector portfolios is denoted by the $M \times 1$ vector $R = (R_1, \ldots, R_M)'$. Its distribution is provided in the following proposition:

**Proposition 1** Under regularity conditions given in the appendix, $R$, the gross return received at date 2 per dollar invested at date 0, is distributed multivariate normal conditional

---

$^{10}$This is with little loss of generality since banks as modeled here face other regulatory constraints which keep their premium small.
on $I_0$ and $I_1$, the information available at date 0 and date 1:

\[ R|I_0 \sim \mathcal{N}[\mu, \Sigma] \quad (4) \]
\[ R|I_1 \sim \mathcal{N}[\mu(1), \Sigma(1)] \quad (5) \]

where,

\[ \mu = \alpha, \]
\[ \mu(1) = \alpha + \beta f(1), \]

**Proof:** See the appendix.

In the proposition, $f(1)$ is a $K \times 1$ vector of factors which represents news about the macro-economy that is learned at date 1; and $\beta$ is an $M \times K$ matrix of factor sensitivities which indicates how different sectors load on the factors. The proof of the proposition relates the return parameters to the structure of the macro-economy and lending markets. For the purposes of the paper, these properties are taken as given to focus the analysis on the interbank market.

Given bank $i$’s equity endowment $E_i$, it chooses its portfolio weights and insured deposits ($\omega_i$ and $D_i$) to maximize the present value of its long-term loans to risk-neutral but uncertainty averse shareholders.\(^{11}\) Note that in this optimization at date 0 the bank ignores the possibility that it may not be able to fund the short-term lending opportunities that arise at date 1. This is rational because the lending and funding shocks are exactly offsetting within the banking system. Therefore, the loans at date 1 will be funded provided the interbank market functions well, which is very likely.\(^{12}\)

For portfolio weights $\omega_i$, and deposits $D_i$, the expected present value of Bank $i$’s long-term loan portfolio to its equity holders at date 0 is $V_i(0)$ given by

\[ V_i(0) = \mathbb{E} \left\{ \frac{\max[(E_i + D_i)\omega_i R - D_i R^D, 0]}{(1 + R^f)^2} \right\} \]
\[ = \mathbb{E} \left\{ \frac{E_i \max[R^D + (1 + L_i)(\omega_i R - R^D), 0]}{(1 + R^f)^2} \right\} \quad (6) \]

where $L_i$ is bank $i$’s leverage ratio given by $L_i = D_i/E_i$, and $R^f$ is the net risk-free rate.

Bank $i$ chooses its optimal portfolio to maximize 6 subject to two regulatory constraints. The first places an upper bound on the banks leverage:

\[ L_i \leq \bar{L}. \quad (7) \]

\(^{11}\)Each bank’s shareholders are assumed to be insiders who know the banks’ own portfolio holdings, but not that of other banks.

\(^{12}\)An additional reason for not hedging in this environment is that it is very costly because hedging involves giving up long-term lending opportunities in exchange for short-term opportunities that are likely to be less lucrative.
The second places an upper bound on the bank’s probability of default conditional on date 0 information $PD_i(\omega_i, 0)$, and thus resembles a constraint on the firm’s credit value at risk:

$$PD_i(\omega_i, 0) \leq \bar{K}.$$  

(8)

The bank will default and become insolvent at date 2, if the returns on its loan portfolio are insufficient to cover its deposits. A little bit of algebra then shows the probability of default is:

$$PD_i(\omega_i, 0) = \Phi\left(\frac{L_i + R^D - \omega'_i \mu}{\sqrt{\omega'_i \Sigma \omega_i}}\right),$$  

(9)

where $\Phi(.)$ is the Standard Normal CDF.

The constraints are an essential part of the bank’s maximization problem. Without them, because of deposit insurance the bank’s objective function is maximized by choosing infinite leverage, and a loan portfolio with infinite variance.$^{13}$ With the constraints, because of limited liability and deposit insurance whose pricing is not risk based, algebra shows given the optimal leverage choice $L_i$, the bank’s optimal asset portfolio has the maximal mean return and maximal standard deviation that satisfies the credit value at risk constraint. Using mean-variance portfolio theory, this implies that bank $i$’s optimal portfolio is on the risky asset mean-variance efficient frontier, and is such that the value at risk constraint binds. For fixed leverage, the set of portfolios for which the credit value at risk constraint binds is an upward sloping line in mean-standard deviation space, and the optimal portfolio $\bar{\omega}$ is where this line intersects the efficient frontier as illustrated in Figure 1.

To model structural uncertainty, I assume that other banks know bank $i$’s leverage and total asset holdings from its balance sheet, but do not fully understand the internal constraints $i$ faces due to limited underwriting capability or other constraints on investment opportunities. This is modeled by assuming that other banks believe $i$ holds a nearly optimal portfolio that lies on an interval between $\omega$ and $\bar{\omega}$ on its mean-variance frontier, and is such that the value at risk constraint binds. Because mean-variance efficient portfolios are convex combinations of each other (provided there are no restrictions on the size of long or short positions), other banks structural uncertainty about $i$ can be expressed as:

$$\omega_i \in \mathcal{C}(\omega, \bar{\omega})$$

(10)

where, $\mathcal{C}(\omega, \bar{\omega})$ is the set of portfolios that are convex combinations of $\omega$ and $\bar{\omega}$.\footnote{Mathematically, $
\mathcal{C}(\omega, \bar{\omega}) = \{\omega : \omega = \theta \omega + (1 - \theta)\bar{\omega}, \ \theta \in [0, 1]\}$.}$^{15}$

$^{13}$In a fully dynamic setting if the bank has franchise value, then this would help to constrain its risk taking somewhat.

$^{14}$There are other ways to motivate structural uncertainty and bounds on $i$’s portfolio weights. The main results in the paper only require that other banks believe $i$’s portfolios is in a closed, bounded, convex set.

$^{15}$Mathematically,
3.2 Date 1

At date 1 banks receive macroeconomic news $f(1)$, and receive stochastic lending and funding shocks. The banks that receive stochastic lending shocks will be indexed by $i$, and those that receive a funding shock will be indexed by $j$. Each stochastic lending shock is a demand for short-term loans from short-term borrowers that have a reservation interest rate of $\bar{R}_l$. The quantity of short-term loans demanded from each $i$-bank has total measure 1. To keep the analysis of the short-term lenders simple we make the following assumptions:

**Assumption 1** Each short-term borrower will default with probability one unless screened by bank $i$, but short-term borrower loans will be risk-free if screened.

**Assumption 2** Each bank $i$ has monopoly power over its short-term borrowers and can thus charge the monopoly rate $\bar{R}_i$ for its short-term loans.

**Assumption 3** The amount of of short-term loans that banks receive or make during date 1 are small enough relative to the scale of their long-term loan portfolios that their default probabilities at date 2 only depend on the performance of their long-run loan portfolios.

Each bank $j$ that receives a funding shock has access to new funds that have measure 1, and that pay an interest rate $R^D$ which is less than the riskfree rate. These funds can be invested at the risk-free rate until period 2, or loaned out in the interbank market to a bank $i$ that needs the funds. I assume the tier of the interbank market in which banks $i$ and $j$ trade together operates competitively. Therefore $R_{i,j}$ the rate on a short-term interbank loan between any bank $i$ and $j$ must be as good as lending at the riskfree rate:

$$R_{i,j} - R^f = \hat{PD}_i LGD_i, \quad (11)$$

where $\hat{PD}_i$ is the perceived probability that bank $i$ will default at date 2 given information that is available at date 1, and $LGD_i$ is the loss given default experienced by bank $j$ if bank $i$ defaults.

Bank $i$ will borrow in the interbank market provided that his borrowing cost is low enough that he can make profitable short-term loans with the funds. This requires that $\bar{R}_i \geq R_{i,j}$, which from equation (11) implies the spread he can charge on the short-term loan must be greater than the spread $i$ pays in the interbank market:

$$\bar{R}_i - R^f \geq \hat{PD}_i LGD_i. \quad (12)$$

The size of the spread $i$ pays depends on how beliefs about $\hat{PD}_i$, are formed, which in turn depends on the trading institutions within the interbank market. There are two
main ways in which trade takes place in the market. The first is bilateral in which one
bank directly arranges a loan with another; the second is when loans are arranged in an
anonymous brokered market. The institutions are treated below and in section 4.

3.3 Pricing in the Bilateral Tier w/ Uncertainty

In this section, I model the bilateral exchange tier of the interbank market in which there
is a bank $i$ that needs a loan from bank $j$, and borrows directly from bank $j$ when bank $j$
is uncertain about bank $i$’s exposures. Like the loan officer in the canonical example, bank
$j$ is risk neutral, but uncertainty averse. Because of $j$’s structural uncertainty over $\omega_i$, he
charges a spread based on his worst case beliefs about $PD_i$ in the bilateral market ($BM$),
given by:

$$\hat{PD}_i(BM) = \max_{\omega_i \in C(\omega_i, \omega_i')} \Phi \left( \frac{L_i + \sum \omega'_i \mu(1)}{\sqrt{\sum \omega'_i \sum(1) \omega_i}} \right) \equiv PH.$$ (13)

This is also denoted as $PH_i$, and will play a role in later sections. If we let $\omega_i^*$ denote $i$’s
true portfolio holdings, and $PD_i(\omega_i^*)$ be $i$’s true probability of default, then the spread that $i$
faces can decomposed into a premium for default and an additional premium for uncertainty:

$$R_{i,j} - R^f = \hat{PD}_i(BM) \cdot LGD_i$$ (14)

$$= [PD_i(\omega_i^*) \cdot LGD_i] + [\hat{PD}_i(BM) - PD_i(\omega_i^*)] \cdot LGD_i$$ (15)

$$= \text{Default Premium + Uncertainty Premium}.$$ 

In some conditions, such as those outlined in Case 1, the news that arrives at date 1
unambiguously shrinks the uncertainty premium:

Case 1: $\forall \omega_i \in C(\omega_i, \omega_i)$

(a) $\omega_i > 0$

(b) $\frac{L_i}{1 + L_i} R^D - \omega'_i \mu < 0$

(c) $f(1)$ is such that $\beta_m f(1) > 0$ for all $m$.

(d) $\Sigma[1] = \rho \Sigma$ for some $\rho \in (0, 1)$

The conditions for premia to shrink essentially require lower volatility, and require that
the expected return on assets that bank $j$ believes bank $i$ must hold in positive amounts
increases. These low volatility, positive news conditions are of the sort that prevailed ahead
of the global financial turmoil that began in March 2007, and may help to explain how spreads could have gone down so much during those times.

Conversely as in case 2, if the bank could have a positive or negative exposure to some sector (such as sector 1 below), then if the news about the expected return in that sector is bad enough, i.e. bounded sufficiently far below 0, it will increase the uncertainty spread for bank i.

Case 2: (a) \( \omega_i[1] \in [-A, A] \) for some \( A > 0 \).
(b) \( \beta_1 f(1) < \psi < 0 \).

If i’s potential exposure is large enough, its uncertainty premium may become too high for it to make loans to its short-term borrowers, causing interbank borrowing between i and j to break down. This generates a welfare loss because the best use of funds is to channel them to bank i’s borrowers.

These results are summarized in the following proposition.

**Proposition 2** Although there is Knightian uncertainty about bank i’s positions, under some economic conditions (such as case 1), favorable economic news can reduce the loan spread that bank i pays for borrowing in the interbank market even though there is Knightian uncertainty over bank i’s positions. Conversely, under other economic conditions, such as those in case 2, sufficiently unfavorable news about some sectors of the market can destroy bank i’s ability to finance its new lending opportunities in the interbank market, effectively causing the interbank market to break down.

**Proof:** See the appendix.

The proposition shows that the uncertainty spread can be low in the right economic conditions. As shown in Figure 2, reproduced from the author, forthcoming (2009), provided volatility is low enough or the expected return on its loans are high enough (not shown), i’s uncertainty premium can remain small even when its leverage is high. However, if leverage is high, and volatility increases, uncertainty premia can increase very sharply, as they did during the crisis.\(^{16}\)

The magnitude of the uncertainty premium in the anonymous brokered market is more complicated because it depends on how worst case beliefs are formed over the set of possible banks with whom a bank may transact. We turn to this topic in the next section.

---

\(^{16}\)The model as currently formulated, does not allow volatility to increase at date 1. However, it is straightforward to extend the model to allow for different volatility regimes at date 1 where volatility in each regime \( s \) is \( \rho(s) \Sigma \) where \( \rho(s) \) is a scalar. In this formulation, the returns as of the information at date 1 can be modelled as Gaussian, and the returns on the banks portfolio as of date 0 are a mixture of Gaussian distributions. All of the results in the paper can be generalized to the mixture setting, and they all continue to hold.
4 Multiple Banks and the Interbank Market for Federal Funds

The analysis for multiple banks is broken into five subsections. Section 4.1 describes the multi-tiered structure of the Federal Funds market; section 4.2 presents a model of the top tier of the market, which is also referred to as the center of the market. In the context of an example, section 4.3 studies how trade in the top-tier can breakdown and examines when a government policy that inspect banks and reveals information about their health and positions can improve welfare. Having illustrated how the top tier of the market may sometimes break down, section 4.4 examines the structure of the top-tier in more detail and shows that the top tier of the market has evolved to have a structure that substantially reduces the likelihood of a breakdown in this market due to Knightian uncertainty over risk exposures. The key that avoids breakdown is the information environment in the market which is determined by who can participate, and what they know about each other. Government can play a role in shaping the information about market conditions that is available, which is the subject of section 4.5.

4.1 The Interbank Market

This section describes the features of the Fed Funds market that are salient for our analysis. Formally, the interbank market is a market for overnight or longer-term lending in which banks trade with each other to fulfill liquidity needs, meet central bank reserve requirements, and finance new lending. Three facts about the market are important for our analysis:

1. Many loans are based on repeated relationships in which banks that have a tendency to borrow or lend to each other on a repeated basis face lower rates given other observable variables that are correlated with their credit risk [Furfine, (2001) for the U.S., Cocco et. al. (2009) for the interbank market in Portugal].

2. Many loans take place through an anonymous brokered market in which the borrowers identity is disclosed to a lender only after a match is established at an agreed upon rate:

“While borrowers and lenders may arrange trades directly with one another, larger more sophisticated market participants tend to arrange most of their trades through brokers. A key feature of brokered trading is that trades are initiated anonymously between interested parties, as a borrowers identity is disclosed to a lender only after a match is established at an agreed-upon loan rate. After a match is established and the lender accepts to lend to the borrower (a decision usually conditioned on the presence of a predetermined credit line between the two parties), the trade is deemed ’executed’ by the broker.” [Bartolini et. al. (2008)]
3. The market has a multi-tiered structure in which small banks are net lenders to medium-sized banks; medium-sized banks in turn are net lenders on average to large banks; while large banks primarily extend loans to each other [Allen et. al. (1989), Furfine (1999), and Bech and Atalay (2008)].

The first two facts about the market, that loans are based on relationship lending, and that loans are made in an anonymous brokered market, are nearly inconsistent with each other. The reconciliation comes from the third-fact that the Federal Funds market is multi-tiered which allows some participants to transact through an anonymous brokered market and others transact based on relationship lending.

Bech and Atalay use network modeling techniques to describe the flow of funds in the market. They note that the center of the market consists of a set of large banks who tend to purchase fed funds from small banks, and then lend the funds among themselves, or to banks outside of the center. The banks in the center of the market are large banks that had on average about 400 billion dollars of assets on their balance sheets at the end of 2006.

I model the interaction of the large banks in the center of the market in the next subsection.

4.2 Banks at the Center

This section models the interaction of the multiple large banks that trade in the center of the Fed Funds market. The center is also referred to as the core of the market, and large banks are core banks. For simplicity, at date 1 each large banks portfolio is modeled as appearing identical to outsiders, which means that all large banks publicly report the same balance sheet leverage; and each banks portfolios are believed to be in $C[\omega_1, \omega_2]$. At the beginning of date 1, all banks learn the value of $f(1)$. The remaining events that occur at date 1 are broken up into $M$ sub-periods, $m = 1, \ldots, M$. In each subperiod, $m$, $N$ randomly selected banks are hit by a lending shock that gives the bank the opportunity to make a short-term loan that matures at date 2. The other $N$ banks are hit by positive funding shocks. As in the earlier analysis, following the shocks, banks that are hit with positive funding shocks can make interbank loans that mature at date 2, or they can invest the funds at the risk free rate until the same date. Banks with positive funding shocks are not allowed to hoard funds across subperiods; this assumption is made for simplicity and does not qualitatively change the results. After these decisions are taken, the sub-period ends, and the next sub-period begins.

Trade in the center of the interbank market is modeled as a random matching game. Each large bank that needs funds places a bid for the funds in the market. The other large banks

---

17The center of the market in my terminology is known as the GSCC or giant strongly connected component of the network. Any bank in the GSCC can channel funds to other banks in the GSCC through either direct links (A lends to C) or indirect links (A lends to B who lends to C). Banks that only lend funds to banks in the GSCC, but cannot borrow from these banks are part of the GIN, or giant in-component. Banks that borrow from the GSCC, but do not lend are part of the GOUT, or giant out component.
that have funds to lend are then randomly matched with the bidding banks. If the lending bank consents to the transaction at the agreed upon rate, then a trade is consummated. For simplicity, if the trade is not consummated, then the borrower and lender do not get the opportunity to again trade in the subperiod. The random matching game closely resembles anonymous brokered trading in the Fed Funds market. The game as specified here gives all of the bargaining power to the borrowing banks. Each borrowing bank captures maximal surplus by proposing to borrow at the lowest possible rate that could be acceptable to lending banks, which means proposing a spread equal to $\hat{PD}_iLGD_i$. Since $LGD_i$ is identical for all banks, this is equivalent to borrowing banks proposing an estimate of $\hat{PD}_i$ that is equal to borrowing bank’s assessment of the worst case probability of default that lending banks could arrive at when making a loan in the anonymous brokered market. Since all borrowing banks will propose the same estimate of $\hat{PD}_i$, randomly matching borrowing and lending banks is an equilibrium in the brokered market. A core borrowing bank may be able to get an even better rate if it can show through more transparency or signalling that it holds a low risk portfolio; it can then transact in the bilateral tier of the Fed Funds market. The fact that core banks choose to transact in the anonymous brokered tier suggests that the costs of further signalling and transparency often outweigh the potential benefits. Equation 10 can be interpreted as the amount of structural uncertainty about individual large banks portfolios that remain after they have signalled to their optimal extent.

When a bank lends in the anonymous brokered market, the bank its lends to is a random draw from the distribution of the set of all banks to whom it may extend a loan. Since lending banks do not know who is on the other side of the market, it could be any of the other $2N - 1$ core banks. In computing the worst case distribution of the credit quality of borrower banks, the lending banks must take into account everything they know about the characteristics of the borrowing banks. Since the borrower banks are ex-ante identical, they all have the same leverage $L_i$, and the same equity capital $E_i$, and assets $A_i$. Lender banks also know that each borrower banks portfolio weights lie in the range specified by equation 10, and we assume they know the aggregate supply of loans that all banks in the center of interbank market may have, denoted by the aggregate endowment vector $Y_M$, but they do not know how these exposures are divided among the banks. In later sections we study the consequences that coarser information about $Y_M$ may have.

Now we turn to the spread that borrowing banks propose. Since lending banks are uncertainty averse, borrowing banks propose a spread that is equal to the worst case probability of default that a lending bank could have. Since each lending bank $j$ has a probability $1/(2N - 1)$ of being randomly matched with one of the other $2N - 1$ banks its worst case beliefs about the probability that its borrowing bank in the anonymous brokered market (ABM) will default is the solution to:

\[ \hat{PD}_i = \left( \frac{1}{2N - 1} \right) \hat{PD}_i \]

\[ LGD_i = \left( \frac{1}{2N - 1} \right) LGD_i \]

---

18It is not clear precisely how much trading large banks in the top tier of the market conduct through brokers. Bartolini et. al. (2008) claim that the largest and most sophisticated participants in the market trade through brokers, while based on conversations with experts Ashcraft and Duffie (2007) suggest that less than 27% of trades by large banks is conducted this way.

19If banks have different $L_i$, $E_i$ and $A_i$ this would complicate the analysis, but have little effect on the main results.
\[
\overline{PD}(ABM) = \max_{\omega_k, k=1,\ldots, 2N} \frac{1}{2N} - \frac{1}{2N-1} \sum_{k \neq j} PD_k(\omega_k) \quad (16)
\]
such that,
\[
\sum_{k=1}^{2N} \omega_k A_k = Y_M; \quad (17)
\]
and for each bank \(k, k = 1, \ldots 2N\),
\[
\omega_k \in C(\omega, \overline{\omega}). \quad (18)
\]

In the above maximization, the borrowing bank computes the worst-case beliefs about default probability that a lending bank could have about its counterparties while accounting for the common knowledge that all banks have. This common knowledge is the adding up constraint that beliefs about bank’s individual loan portfolios must add up to the aggregate loan supply in each sector, which is known (equation 17); as well as the constraint that each bank’s portfolios lie within an interval on the efficient frontier (equation 18).

In concrete terms each lending bank’s decision problem is analogous to a consumer randomly drawing an apple from a barrel and eating it. Before doing so, based on what the lending bank knows it contemplates the possible prior distributions of apple quality in the barrel, where each possible prior distributions is characterized by both the proportion of good and bad apples, and the quality that each good and bad apple could have. When the lending bank is uncertainty averse, the amount it pays for a randomly chosen apple, or the amount it charges for a randomly chosen loan is based on its worst case beliefs about the possible distributions from which it is drawing.

Recall that in the anonymous brokered market, it is the borrowing bank that proposes the spread paid by lending banks. In doing so when each borrowing bank computes the worst case distribution of borrowing bank quality that a lending bank could face, it solves the constrained problem while taking into account what the portfolio holdings of the lending bank could be, since from equation 17, the lending bank’s portfolio constrain beliefs about the borrowing banks portfolios. Because equations 17 and 18 are satisfied by banks initial holdings, there is a solution for the worst case probability of default. To find the solution, I make one more mild auxiliary assumption:

**Assumption 4** For each bank \(k = 1, \ldots 2N\), for every feasible \(\omega_k\), \(PD_k(\omega_k) < 0.5\)

This is a very mild assumption except for extremely severe banking crisis. As illustrated below, the purpose for the condition is that it guarantees the objective function in equation 16 is convex, which simplifies the analysis.
To solve for the worst-case beliefs, we note two facts. First, the constraint set is convex. Second, in the proof we show that the objective function is a convex function. When a convex function is maximized with respect to convex constraints, the solution is at the extremes. This means if $PD(\omega) > PD(\overline{\omega})$ then the solution for worst case beliefs involves believing as many potential borrowing banks as possible hold portfolio $\overline{\omega}$, subject to the other constraints of the maximization. Importantly, when equation 17 holds as an equality, then the worst case beliefs involve the others holding $\omega$ and no more than one bank holding a portfolio in the interior of $C(\omega, \overline{\omega})$ since if more than one did, by convexity $\hat{PD}_i$ could be increased by moving one portfolio up toward $\overline{\omega}$ and the other toward $\omega$, which violates the condition that the original beliefs about worst-case portfolios was an optimum.

A comparison of $\hat{PD}(ABM)$ and $\hat{PD}(BM)$ shows how the two markets handle uncertainty about asset holdings slightly differently. To make the comparison slightly easier, it is useful to slightly perturb the beliefs about worst-case asset holdings in the anonymous brokered market so that the one bank that holds a portfolio that is not $\omega$ or $\overline{\omega}$ instead holds whichever of the two portfolios is associated with the highest PD. In the analysis below we treat the portfolio that has the highest PD based on date 1 information as if it is $\overline{\omega}$. This is without any loss of generality because all the results go through if it is the other portfolio instead.

With the perturbed holdings, worst case beliefs in the anonymous brokered market about the default probability of a randomly chosen bank are that it will default with a probability that is no greater than $\theta P^H + (1 - \theta) P^L$ where $P^H$ is the default probability associated with the highest default probability portfolio (say $\overline{\omega}$, and $P^L$ is the default probability associated with other boundary of banks constraint sets ($\omega$), and $\theta \in [0, 1]$ is the proportion of banks that hold the high risk portfolio. The result follows because the perturbed portfolio holdings are infeasible (they violate the constraint in equation 17) and have a slightly higher probability of default than is possible from a randomly chosen bank in the anonymous brokered market.

Recall that $P^H [= \hat{PD}_i(BM)]$ is the worst-case probability of default in the bilateral market when there is uncertainty. An immediate implication is that worst case probabilities of default in the anonymous brokered market are lower than those in the bilateral market with uncertainty. Formally,

**Proposition 3** Under assumption 4, and the constraints on beliefs from equations (17) and (18), when there is uncertainty, the spreads that banks pay in the anonymous brokered tier of the interbank market are less than or equal to the spreads they would pay when borrowing in the bilateral tier of the interbank market.

**Proof:** See the appendix.

The proposition shows that trading with a single bank in the bilateral market when there is uncertainty generates higher spreads than when dealing with a randomly matched bank in the anonymous brokered market. The intuition is that any single bank could hold the worst
case portfolio, but it may be impossible for all of them to do so because it may violate
the adding-up constraint (equation 17). Therefore, if brokers act to ensure random matching in
the anonymous brokered market, then worst case beliefs must account for the randomness
of the matching, which then leads to lower spreads than in the bilateral market.

An implicit assumption in the above reasoning is that if a bank is approached to lend
in the bilateral market, then the arrival of the borrowing bank is not treated like a random
arrival. This can be rationalized by noting that banks may not have well formed beliefs about
arrivals to borrow outside of the brokered market, and thus treat unanticipated arrivals in
the bilateral market suspiciously and charge a higher spread in response. This treatment is
analogous to how non-random arrivals are often treated in other real world situations. For
example, when asking for directions in a crowded train station, it may be fine to ask for help
from almost any randomly chosen person, but if a person approaches to offer directions, then
that help should be treated more skeptically because of uncertainty over the persons motive
in having arrived to offer directions.²⁰

It is important to emphasize that the results in proposition 3 only apply for borrowing
banks when lending banks are uncertain about the borrowing banks portfolio composition,
and when the range of uncertainty about individual banks portfolios is approximately the
same, as it is in the proposition. These conditions are more likely to be satisfied for large
banks because it is likely to be very expensive for them to reveal their portfolios; and
without good information on portfolio holdings, beliefs on portfolios are more likely to be
dominated by investment opportunity sets, which are more likely to be similar for large
banks. Therefore, the results in proposition 3 help to explain why large banks tend to trade
with each other in the anonymous brokered market. The results do not however explain two
other facts about the interbank market:

1. Why small banks do not borrow in the anonymous brokered tier of the market?
2. Why is there relationship lending in the bilateral market?

Answers to these questions are delayed until section 4.4, which studies the structure of
the interbank market. The next section takes the structure of the top tier of the market as
given, and then studies market breakdown.

4.3 Market Breakdown and Government Information Provision to
Restart Markets

With this solution for worst-case beliefs in hand, the analysis now focuses on illustrating
how the anonymous brokered tier of the interbank market can break down, and why private
efforts to restart it may fail while government efforts to restart it may succeed and improve

²⁰The train station analogy is based on a comment by Pete Kyle.
welfare. The ideas for why private efforts may fail and government efforts may succeed are general, but to illustrate them, the analysis is specialized so that there are only two sectors. Loans to a pool of borrowers in sectors 1 and 2 will be referred to as assets 1 and 2. Core banks hold $Y_M[1]$ of asset 1, and each bank $i$ has the capacity to hold the entire stock of that asset ($\bar{w}_iA_i[1] > Y_M[1]$).

If the news that arrives at date 1 is sufficiently bad for asset 1, then because of convexity, and our assumptions on capacity, the worst-case beliefs that maximize $\hat{PD}_i$ will involve one borrower bank holding all of asset 1 (and potentially some of asset 2), and the other $2N - 1$ banks holding no exposure to asset 1. This is equivalent to having the belief that there are $2N - 1$ banks with a low probability of default, $P_L$ and one bank with a high probability of default $P_H$. This implies:

$$\hat{PD}_i = P_L + \frac{1}{2N-1}(P_H - P_L).$$

(19)

From equation 12, we know that if the worst-case beliefs $\hat{PD}_i$ exceeds some maximal level, denoted $\hat{PD}_i$, that financing costs for short-term loans will be too high to justify extending these loans, and hence the interbank market may break down. Assume that the news $f(1)$ is sufficiently bad that it does break down.

One possible solution would be for good banks to signal their quality by revealing information about their portfolios. For simplicity, assume the information fully and credibly reveals the bank’s portfolio. In addition, assume the bank can borrow at a rate commensurate with its true credit quality after signalling. Suppose the cost of signaling in this way is $c$, given by

$$c = (\bar{R}_i - R^D - P_LLGD_i) \times \frac{M}{2} + \epsilon,$$

(20)

for some $\epsilon > 0$. This cost is then equal to the expected surplus a good bank could hope to earn by signaling plus a little bit, which means for this $c$, no bank would be willing to signal on its own because it could not pass the costs of signalling on to its borrowers. In this case, the interbank market will fail because of uncertainty, and private incentives to restart it by signalling will be insufficient.

It turns out that if government pays to acquire the information on banks risk exposures, it can improve the function of the interbank market and create social surplus. To illustrate the mechanism for improving surplus, suppose one bank has default probability $P_H$ and the other $2N - 1$ have default probability $P_L$, i.e. suppose that the worst case beliefs associated

\footnote{By convexity $P_H < 0.5$.}

\footnote{If information is gathered, $P_L$ is the lowest possible risk that bank $i$ could possibly have. Since its true $PD$ will be greater, with this information cost the loan will not be extended.}

\footnote{Note that in the present treatment government actions only create surplus by generating information and changing agents beliefs. In Caballero and Krishnamurthy (2008), government actions create surplus through an additional channel: welfare is measured relative to beliefs of a benevolent social planner, rather than relative to the beliefs of the agents in the model. We rule out this additional channel in our analysis.}
with uncertainty are correct. If the government searched for the one bad bank sequentially, publicized for each bank searched whether it is H or L and announced its risk exposures, and then stopped after the bad bank was found, its expected search costs are \((2N-1)c/2\). After the bad bank is located and its presence is announced, then the other \(N-1\) low risk borrower-banks can borrow in the interbank market and then extend short-term loans. Across all \(M\) subperiods, assuming the high risk bank is shut, leaving one bank without an ability to fund its short-term loans, manipulation of equation 20 shows this will generate loans with surplus \(2(N-1)(c-\epsilon)\) before information costs. Which means that after expected search costs, the social surplus generated by the government information provision is on the order of \(Nc > 0\), showing government sponsored sequential search and release of information can improve social welfare and the functioning of the interbank market in some circumstances when private incentives are insufficient. The same logic applies if the government’s costs of revealing information are higher than that of the private sector; and as we show below there are special circumstances in which the surplus generated by government action can be arbitrarily large.

These results are formalized in the following proposition.

**Proposition 4** There exist \(P_H, P_L, N, M, c, LGD_{i,j} \) and \(\bar{R}_l\) such that:

1. Because of uncertainty about banks’ risk exposures the interbank market may break down.
2. Private provision of information on the exposures may be too costly to restart the market.
3. Sequential government-supported inspections in which the government inspects banks, and announces their health and risk exposures may restore market functioning and improve social surplus. This may be possible even if the government faces higher costs of information gathering than the banking sector faces.
4. There are parameter values for which results 1 - 4 hold, and for which the expected social surplus from sequential government-supported inspections can be made arbitrarily large.

**Proof:** See the appendix. □.

The novel aspect of the proposition is that public provision of information on exposures is welfare improving when private information on exposures will not be provided. Private provision of information fails because the private costs of signalling that a bank is high quality are too high to be passed on to banks’ short-term borrowers. Therefore, high quality banks will not signal they are high quality — and of course bad banks will not pay the costs to signal that they are low quality.

Public provision of information is welfare enhancing because when public search for bad banks succeeds, the knowledge that a weak bank has been located and closed, and the

---

\(^{24}\)Because the banks are identical based on observable characteristics, this search strategy is optimal. 20
knowledge of its positions combined with knowledge of the outstanding assets \( Y_M \) allows banks to update their assessment of the worst-case risks of other banks, reducing economy-wide uncertainty, and creating an external benefit. The way the example is parameterized, for the expected external benefit to be positive, the government’s search activity has to stop before every bank is examined because if every bank is examined the costs of searching would just be equal to the total private search costs, and the total surplus would be negative, as it is in the private case. Because on average the government only has to search in half of the banks, government search is welfare enhancing.

The proposition is proved in the special case where worst case beliefs on asset exposures and the exposures coincide. In a more general setting, the worst case beliefs will be the same, but the actual asset exposures will differ. As a consequence, the surplus that is generated by the government’s search strategy will depend on how the assets are allocated, and the government’s stopping rule for deciding when its bank inspections have located enough bad assets. To analyze how the distribution of risky assets across banks affects the surplus calculation I assume that government searches for risky asset exposures one bank at a time, and stops searching when the market restarts and surplus is at least 40\% of what it would be if the risky assets were evenly dispersed among all banks. This search rule is not optimal, but was chosen so that search would continue beyond the point when interbank markets just restart, since more search is typically beneficial in those cases. However, search is not allowed to continue too long when the risky asset holdings appear to be very dispersed since in those circumstances more search is costly. To analyze the more general setting requires simulation.

**Simulation Analysis**

In the simulations there are 2 assets (sectors), and \( c, 2N, R_l \), and the aggregate exposure vector of core banks \( Y_M \), are such that the worst case beliefs about \( P_H \), and \( P_L \) satisfy the properties in proposition 4. This means \( \overline{PD} \) is such that the interbank market breaks down initially, and \( c \) is such that the surplus would be as in the proposition if the most pessimistic beliefs were actually consistent with the true allocation of risky assets among the banks.

There are \( 2N = 20 \) banks that are ex-ante identical, but their portfolios are tilted by different amounts toward the high risk asset (asset 2), which can be interpreted as the high-risk real-estate sector. In particular, each bank’s portfolio weight in risky asset 2, \( w_2 \), can range from 0.4 to 0.6, and its weight in asset 1 is \( 1 - w_2 \).

Given the asset supplies, there is a \( 2N \times 2 \) matrix of portfolio weights \( W_d \) (with the first and second columns corresponding to weights in assets 1 and 2) in which the holdings of asset 2 are maximally dispersed (all

\footnote{It would of course be trivial to create examples where the government faces lower costs of signalling bank quality than the banks themselves. This could occur, for example, if the banks’ attempts to signal their own quality are not credible, but that government signals are credible.}

\footnote{For simplicity, \( \epsilon \) in the proof of the proposition is set to 0.}

\footnote{The book value of each banks long-term asset portfolio is normalized to 1 (\( A = 1 \)). In addition, \( R^D = 1.03 \), assets 1 and 2 have standard deviations of return conditional on \( I_1 \) given by .015 and .06, and the assets returns have correlation 0.3. For any simulations in which the bank defaults and becomes insolvent, the loss given default experienced by its short-term lenders is 50\%. The expected returns \( \mu(1) \) on assets 1 and 2 were allowed to differ depending on economic conditions.}
banks hold the same amount of asset 2), and a second matrix $W_c$ in which the holdings of asset 2 are maximally concentrated (one bank has portfolio weight $w_2 = 0.6$, and the others have $w_2 = 0.4$). The simulations are conducted for matrices of portfolio weights $W(\lambda)$ that are convex combinations of the maximally dispersed and maximally concentrated holdings:

$$W(\lambda) = \lambda W_c + (1 - \lambda)W_d, \quad \lambda \in [0, 1],$$

where $\lambda$ can be interpreted as a concentration index that ranges from 0 (maximally dispersed) to 1 (maximally concentrated). For each matrix, the government’s inspections are random searches that stop when the surplus created from the government’s actions is high enough, as discussed above. To estimate the expected surplus generated for each matrix, 10,000 random searches are conducted, and the average surplus generated net of search costs is reported.

The results from our baseline simulation are reported in Figure 3. The figure shows that the expected surplus generated by the government’s search policy depends on how the holdings of asset 2 are concentrated. The case of maximal concentration corresponds to the situation in the proposition, and yields the maximal expected surplus. When the exposure is more dispersed, the surplus is lower, and in the extreme when it is very dispersed, the surplus drops below 0 (Figure 3). The reason that concentration matters is that when exposures are dispersed, a large number of inspections are required to restart the interbank market—and each inspection is costly. Conversely, when exposures are concentrated, once the large exposures are identified, the interbank market quickly restarts and less surplus is exhausted on costly search and inspection. It should be added that there is also a dependence on the exact criterion for restarting search. In simulations that are not shown, I considered the possibility of stopping the search process after the interbank market restarted. With the exception of very concentrated asset holdings this often generated very little surplus because after a sufficient amount of risky asset is located the interbank market restarts, but with interbank spreads that make the banks nearly indifferent between making loans or not, thus generating little surplus. Continuing to search beyond the point when markets restarted generated more surplus in most cases. Overall, the analysis shows that the government search strategy generates positive surplus for many, but not all asset concentrations.

An important assumption in the analysis of this example is that the government’s ability to assess a banks’ risk is just as good as that of banks themselves. A more realistic assumption is that government has to pay higher costs to assess risk in the banking sector than banks’ themselves. To analyze the role of different costs for the government, we assumed that the government’s costs of evaluating banks range from being the same as the banks cost to 3x that amount. The analysis based on this range of costs shows that government intervention raises surplus when the asset holdings are concentrated enough and costs are low enough, but that there is a very broad range of concentration and costs of which the surplus generated by the government policy is negative (Figure 4).

---

28Note: in these simulations I have relaxed the assumption that one bank can hold all of the high risk asset in its portfolio since that assumption is not essential. For comparability purposes with proposition 4, the condition I did maintain is that $Y_M$ is such that worst case beliefs involve one high risk bank and $2N - 1$ low risk banks.

29Recall the banks are monopolists in lending to final borrowers that have inelastic demands for funds, so the total surplus is just the banks surplus.
One of the reasons the surplus is negative is that government search is completely random in the example which is equivalent to assuming that the government has no information about banks relative exposures to the distressed asset class. If we instead assume that government has prior information about the relative risk exposures, then using that information has a dramatic effect on the surplus calculations. In particular, under the strong, but not unrealistic assumption that government knows the ranking of banks exposures to the distressed asset class, inspecting banks by ranking has a major effect on the surplus calculations, generating positive surplus for a very wide range of concentrations, and even for high government search costs (Figure 5).

In addition to the simulations with dispersed asset holdings, I also generalized proposition 4 to allow more than one bank to have a high default probability, while the others to have a low default probability. When there are $2N = 20$ banks and up to five banks have a high probability of default, the results in the proposition generalize. It generalizes for some situations when $N$ is larger, but they have to be checked by simulation or very tedious combinatorics so I have not done a complete analysis.\(^{30}\) When $2N = 20$ if for worst case beliefs the number of high risk banks exceeds 5, the expected number of random searches required becomes too large to generate positive surplus.

In sum, the findings on the government’s search and information provision policies show:

1. The external benefits of the government revealing information on bank health and risk exposures may justify a government search and information provision policy in order to restart markets.

2. Government search and information provision policies are far more effective and less costly if the government has prior information on the relative size of core banks exposures and uses this information to prioritize the order in which it searches banks and reveals information about their risk exposures.

3. The basis for the government’s policy, as formulated here, is very narrow. There may be a broader basis for government search and information provision beyond the considerations in the model.

Regarding point 3, there are two additional potential justifications for government intervention. The first is the government’s costs of learning information about banks and then revealing it to reduce uncertainty may be lower than the costs to the banks themselves because banks may have a severe credibility problem when trying to signal during a financial crisis. Second the surplus calculations may undercount surplus because they assume banks capture all surplus from their lending activity, when frictions in lending markets (such as adverse selection) may prevent banks from capturing some surplus.

\(^{30}\)To prove the result for $2N = 20$, I assumed the government searched until all bad banks were found, and computed the expected number of searches required using Monte Carlo analysis. The other parts of the proof are similar to the proposition.
In closing this subsection, it is useful to compare the government intervention on information release policy proposed above with past historical interventions that provided information on bank health. Park (1991) found that the Bank Holiday of (1993), which temporarily closed banks, and only reopened those deemed solvent, and earlier experiences with bank clearinghouse’s that issued clearing loan certificates during a crisis both improved market functioning because it provided the public with information about the quality of bank assets, which allowed them to distinguish between solvent and insolvent financial institutions.

More recently, the Supervisory Capital Assessment Program (SCAP) in the United States has been perceived as having led to an easing of credit conditions because it also provided a clearer picture of bank’s financial health by subjecting banks portfolios to stress tests, and then releasing the results of the exercise.

One of the findings revealed by the stress-tests was that within the pre-crisis categorization of loans for regulatory reporting purposes, there were significant differences in loss rates within asset categories31:

“For example, while the median two-year loss rate on first-lien mortgages was 8 percent across the 15 Bank Holding Companies (BHCs) with a material amount of mortgages, the rates varied from a low of 3.4 percent to a high of nearly 12 percent. For second and junior lien mortgages, the range among 14 BHCs was 6 percent to 21 percent, and a median rate of about 13 percent. Such variation reflects substantial differences in the portfolios across the BHCs, by borrower characteristics such as FICO scores, and loan characteristics such as loan to value ratio, year of origination, and geography. These differences result in significant variation in loss estimates at the firm level as compared with applying a single loss rate per asset category to all BHCs.32

The differences in loss rates across the categories show that the categorization of risk exposures that may have provided sufficient certainty about bank’s risk pre-crisis, may have been too coarse when economic conditions deteriorated. In this interpretation, the contribution of the stress test is that it revealed information on banks risk-exposures to subcategories of loans that varied by borrower quality. This helped reduce uncertainty over who was exposed to what, and helped bring down borrowing spreads.

In sum, theory and past experience both show there may be a need for government to occasionally step in and improve the quality of information in markets. There is also a case for encouraging the development of market institutions that reduce the effects of uncertainty ex ante. In the next subsection I discuss how the structure of the Fed Funds market serves that role, and then in the following subsection I discuss the role for government to further reduce uncertainty in that market.

31 “For the most part [in the SCAP exercise], these categories [for assets] are based on regulatory report classifications to facilitate comparison across BHCs and with information reported by BHCs in their regulatory filings [The Supervisory Capital Assessment Program: Design and Implementation (2009), page 8].”

32 “The Supervisory Capital Assessment Program: Overview of Results (2009), page 10.”
4.4 Uncertainty, Information, and the Structure of the Interbank Market

This section studies why the interbank market has a multi-tier structure with the features that small banks do not borrow in the anonymous brokered tier of the market, but can engage in relationship borrowing arrangements with large banks. A fully formal analysis is beyond the scope of this paper, but an informal analysis follows. In the analysis small banks are added to our basic framework, and then it is extended in four ways.

First, small (S) and large (L) banks have a signalling technology, which if costs $C(S)$ and $C(L)$ are paid, perfectly reveal the bank’s portfolio composition. At date 1, banks that receive a signals pay a one-time costs $e(S)$ and $e(L)$ to analyze another bank’s signal. Because large banks are usually more complex than small banks, $C(L) > C(S)$; and $e(L) > e(S)$. I assume banks can choose to pay these costs and signal at the start of date 1 just after information has arrived.

Second, the choice of counterparties in the anonymous brokered market is modelled explicitly. In particular, trading in the anonymous brokered market is modelled as a multi-stage game. The first stage occurs at date 1, after banks have signalled or not. In this stage each small and large bank provide the broker with a list of banks to whom it is willing to lend in the anonymous brokered market. For simplicity these lists are treated as public information. After the first stage, but before the second, trade occurs in the bilateral market. The bilateral trades can generate a need for funds that will be fulfilled in the anonymous brokered market; more details on this will be provided below. After trade occurs in the bilateral market, the second stage begins. In the second stage, M subperiods of trade occur. In each subperiod orders to borrow arrive; the orders are randomly selected and distributed by the broker to one of the banks that have have that borrower on their list. The borrower offers a price; and the lender decides whether to accept or not. If a borrower is declined; the broker throws his order back in the pool of borrowers and he can borrow again in the subperiod if selected. Lenders are assumed to be unable to lend again within a subperiod if they turn a borrower down. They are assumed to invest their excess funds from a subperiod at the risk-free rate until date 2.

Third, market conditions differ in two ways. First, lending banks earn a small amount of economic rents from borrowing banks. In particular, I assume lenders engage in a form of Bertrand competition, but with price discreteness that prevents lenders from competing away all rents. Second, I assume that there are more banks that experience positive cashflow shocks than experience positive lending shocks. This makes the market a buyers market, which simplifies the analysis.

Fourth, the bilateral and anonymous brokered markets are modeled jointly. Recall that there are M subperiods of trade at date 1. In each subperiod trade in the bilateral market takes place first, and then trade in the brokered market. In the bilateral market, the lending

---

33 Although small banks are net lenders in the Federal Funds market, Furfine (199) finds that they are not excluded from borrowing in the market.
The extended framework is used to suggest an answer to the questions:

1. Why small banks are excluded as borrowers from the anonymous brokered market?

2. Why is there relationship borrowing in the bilateral market?

The answer to the first and second questions are best explained in the context of the example with two risky assets that was presented in section 4.3. To build on this example, suppose that in addition to the \( N \) large banks that receive lending shocks and attempt to borrow in the anonymous brokered tier of the interbank market there are also \( S \) small banks, \( s = 1, \ldots, S \), with identical observable characteristics (assets \( A_s \) and total loan supply vector \( Y_s \)) that could attempt to borrow in the top tier of the market. In this case the worst case probability of default in the brokered market is given by:

\[
\hat{PD}(ABM) = \max_{\omega_k, k = 1, \ldots, 2N - 1, \omega_s, s = 1, \ldots, S} \frac{1}{2N - 1 + S} \left( \sum_{k=1}^{N} PD_k(\omega_k) + \sum_{s=1}^{S} PD_s(\omega_s) \right) \tag{21}
\]

subject to the constraint that:

\[
\sum_{k=1}^{2N} \omega_k A_k + \sum_{s=1}^{S} \omega_s A_s = Y_M + Y_s, \tag{22}
\]

where \( \sum_{s=1}^{S} A_s << Y_M[1] + Y_S[1] \).

This is a case where the size of small banks collectively is much smaller than the amount of loans that have been made to sector 1, which is a sector of the economy whose loans are expected to perform poorly conditional on date 1 information. Since each bank’s probability of default is increasing in the amount of assets it holds in sector 1, because of the convexity

\[34\text{In reality lending banks may be able to extract additional rents because borrowing banks want to avoid the signal evaluation costs that would be associated with switching lenders.}\]
of the maximization problem, from the proof of proposition 1, it follows that the portfolio weights which maximize the robust probability of default set \( \omega_s[1] = 1 \) for small banks \( s = 1, \ldots, S \), and then allocates all remaining holdings of asset 1, to a single large bank. As in the two asset example, the other large banks concentrate their asset holdings in asset 2. To see how including small banks in the anonymous brokered top-tier of the interbank market affects robust default probabilities versus the case when they are not included, note that each small bank has a default probability \( P_S \) where \( P_S > P_H \) in the example with two risky assets. On the other hand, the large bank that has an exposure to asset 1, now has default probability \( P'_H \) where \( P'_H < P_H \). Provided that \( N \) is greater than 2 (which is required for an anonymous brokered market to make any sense) and \( S > 2 \) (which is realistic) allowing the small banks to participate in the center of the interbank market increases the worst-case probability of default. This result is summarized in the following proposition

**Proposition 5** If there are 2 or more small banks and two or more large banks that want to borrow in the anonymous brokered tier of the interbank market, then if \( \sum_{s=1}^{S} A_s << Y_M[1] + Y_S[1] \), then the worst-case probability of default for the two asset example in equation 21 is increasing in the percentage of small banks that participate in the anonymous brokered tier of the market.

**Proof:** See the appendix.

Although proposition 5 is specific to the two-asset example, the result that allowing small banks to borrow in the ABM increases worst case probabilities of default is more general, and is driven by two effects. The first effect, “the many small banks effect”, is that many small banks can hold a high risk portfolio while still satisfying the resource constraint (equation 17). Therefore, the inclusion of small banks, at least at first, increases the worst case probability of default by increasing the fraction of banks in the interbank market that could have the highest possible risk.

The second effect, “the diversification effect” is that small banks by virtue of their size can hold portfolios that are less diversified than a large bank could hold. For example, if there is a problem in a “small” 1 billion dollar sector of the economy, then a large bank by virtue of its size (say 400 billion dollars or more) might only be capable of having a small portfolio weight in that sector. Conversely, many small banks may be able to have a high portfolio weight in the distressed sector – and therefore many small banks can have a high probability of default when beliefs are worst case. To formally model the diversification effect, the analysis would have to account for differences in large and small banks abilities to obtain particular portfolio exposures, which goes beyond the scope of this paper.

The result of proposition 5 shows why small banks who have not signalled as borrowers can increase \( \hat{PD}(ABM) \) for a bank that chooses to list small banks as potential borrowers in the anonymous brokered market. Building on proposition 5, I show that there are equilibria of the extended model in which large banks choose to exclude small banks from their lists of

\[ \text{35The more general result is available from the author upon request} \]
borrowers in the ABM. To establish the equilibrium result, suppose first that all large banks exclude small banks from their lists. In the proposed equilibrium, large banks that make interbank loans earn rents from the banks that they lend to, and large banks that lend to short-term borrowers also earn positive rents. To show that the proposed equilibrium is an equilibrium it suffices to show that no large or small bank would gain from deviating.

One form of deviation is that a large bank could add a small bank to his list. If he added small banks to his list who had not signalled, this would raise his $\hat{PD}(ABM)$ and his reservation spread, but in a borrowers market, borrowers will not offer a higher spread because they know brokers will eventually match them with a bank that offers lower spreads. Thus the lender who listed small banks who had not signalled would earn lower profits than if he had not deviated.\(^{36}\)

On the other hand, if a large bank added small banks to its list who had signalled, then small banks who were lower risk than the spreads among large banks in the ABM would not want to participate in the ABM since they could earn lower spreads if there is enough potential competition in the bilateral market. This means only small banks with a relatively high $PD$ would choose to borrow in the ABM, but large banks would not include them on their lists because it would price them out of the market, as explained above. Thus small banks are excluded from the ABM as borrowers.\(^{37}\)

A different type of deviation is that a large bank could signal its quality and attempt to borrow in the bilateral market. If signalling costs are high for large banks, as seems reasonable, many large banks will not be able to do this, and will instead prefer to borrow in the brokered market. In reality large banks differ in their size and signalling costs. This suggests that some large banks that are relatively small be able to signal and then borrow at a low cost — while many others will not signal but will borrow in the brokered market.

This leave open the question of where small banks could lend and borrow. Regarding lending, the analysis suggests that small banks would prefer to lend in the anonymous brokered market for the same reason that large banks would. They often do this in reality, albeit through correspondent banking in which a large bank does the transaction on their behalf in the brokered market.\(^{38}\) Regarding borrowing, the analysis suggests small banks would need to borrow in the bilateral tier of the market — and would have to signal to avoid paying an uncertainty premium. Because signal evaluation is costly, and some of the incidence of the cost of signal evaluation would fall on small banks, each small bank would prefer relationship loans that involve borrowing from one or a small number of lenders.\(^{39}\)

In sum, the analysis for the extended model shows that the multi-tiered structure for

\(^{36}\)More formally, because the source of the rents is price discreteness, if the prices that can be quoted are fine enough, then the lender with a higher $\hat{PD}(ABM)$ would be undercut most of the time.

\(^{37}\)This is consistent with empirical analysis that small banks ability to borrow in the Fed Funds market is constrained [Ashcraft et. al. (2009)].

\(^{38}\)The small bank does not borrow from the large bank acting on their behalf as part of this transaction.

\(^{39}\)Relying on a small number of lenders increases evaluation costs, but creates competition among potential lenders.
borrowing in the interbank market is an equilibrium. In that equilibrium, because signalling is costly for large banks, they trade with each other in the brokered market. This avoids information costs, but keeps uncertainty premia low because of anonymous matching, and also because small banks are excluded as lenders from the brokered market. For small banks, whose costs of signalling are lower, they choose to borrow on a relationship basis.

The spreads that large banks pay each other in the anonymous brokered market depends on both the number of large banks trade in the brokered market, and on the types of assets that get shocked. To analyze the role of each, first consider the role of the number of core banks. Bech and Atalay note that in 2006 banks in the center of the Fed Funds market received funds from 19.1 banks on average and lent funds to 9.3 banks on average. These average numbers can be misleading because the distribution is very skewed. For example, the most active bank in the center received funds from 127.6 banks in 2006, and lent funds to 48.8 banks. This shows the number of banks that are potentially on the other side of trades in the anonymous brokered market is large. The effect of such a large $N$ can be illustrated from the two asset example. When the bad asset can be absorbed by one large bank, then $\hat{PD}(ABM) = P_L + \frac{1}{2N-1}(P_H - P_L)$. As $N$ grows large, with $Y_M$ fixed, this expression approaches $P_L$, which means that a large number of large banks interacting anonymously in the brokered market leads to low spreads when the bad asset (asset 1 in the example) is in one small sector that (with worst case beliefs) can be absorbed by one bank. Suppose instead asset 2 was severely affected and core banks total exposures is much greater than any banks ability to absorb it on its balance sheet $Y_M[2] >> A_k$. Then worst case beliefs imply that many banks ($J > 1$) may have a large exposure to asset 2, which implies a much larger value for the worst case probability of default: $\hat{PD}_i = P_L + \frac{J}{2N-1}(P_H - P_L)$, and a market that is more prone to collapse due to uncertainty. In practical terms, this analysis means the interbank market may be resilient to problems by a major firm (such as WorldCom, GM, or Chrysler), but it may be less resilient when a severe shock hits a sector that many banks are exposed to, such as housing.

**Summary**

In summary, this section has shown that the two-tier structure of the Fed Funds market, and who participates in each tier, helps to reduce the effects that uncertainty about banks positions have on interbank spreads. In particular

- If small banks participate with large banks in the top tier of the interbank market, the small banks participation increase uncertainty premia through a “many small banks effect” and through a “diversification effect”. For this reason, small banks do not participate as borrowers in the top tier of the market.

- When small banks borrow in the interbank market, they do so on the basis of relationship loans from a small number of lenders.
• Many large banks participating in the top tier of the interbank market reduce uncertainty premia, especially uncertainty regarding exposures to small sectors of the economy.

• The top tier of the interbank market can generate significant uncertainty premia when sectors to which many large banks must have exposures suffer.

The next section discusses how the government’s role in shaping the set of publicly available information further reduces information premia.

### 4.5 Role for government in shaping prior information

The model suggests two informational roles for government policy. The first is to provide information that helps to resuscitate markets when they collapse. The second is to shape the information environment to improve market function and reduce the likelihood of future collapses. To illustrate the role for shaping the information environment, consider again the problem of solving for worst case beliefs in equation 16, but suppose banks’ information on $Y_M$, the vector of core banks asset holdings is coarsened. In particular instead of observing $Y_M$, banks only observe $\tilde{Y}_M$, which equals $Y_M$ plus a positive noise vector $\epsilon_{BO}$. Banks information may be coarser than $Y_M$ if published banking statistics do not separately report core banks exposures, or if available information on core banks is not detailed. The coarsened information changes the constraint in equation 17 to

$$\sum_{k=1}^{2N} \omega_k A_k + \epsilon_{BO} = \tilde{Y}_M, \quad (23)$$

The coarsening relaxes constraints in solving for worst case beliefs. For example, if a high value of $\tilde{Y}_M[1]$ is reported, worst case beliefs might erroneously conclude that core banks exposures to asset 1 are greater when in fact non-core banks exposures are actually greater. The result is more precise information on core-banks asset holdings $Y_M$ reduces uncertainty premia, and more coarse information makes it worse. This is stated formally below:

**Proposition 6** When the information on core banks total asset holdings $Y_M$ is coarsened as in equation 23, then, all else equal, worst case beliefs about core banks probability of default increase, and uncertainty premia increase as well. Less coarse information about $Y_M$ has the opposite effect.

**Proof:** Obvious because coarser information about $Y_M$ relaxes a constraint in the optimization problem 16 that solves for worst case beliefs regarding default probabilities. $\square$.

Two corollaries follow immediately:
**Corollary 1**  If banks are uncertainty averse, then all else equal, coarser information on $Y_M$ increases the likelihood that markets will collapse because of uncertainty.

**Proof:** Straightforward since interbank lending collapses when spreads become high enough. □.

**Corollary 2**  If banks are uncertainty averse, then all else equal, coarser information on $Y_M$ will increase the cost of using sequential government inspections to restart interbank markets that collapsed.

**Proof:** If the information is coarser, then after the government inspects a bank, and reveals information on its exposures, the worst case PD of the remaining banks will be higher because coarser information relaxes constraints on their worst case PD. Therefore, the number of searches required to reduce spreads enough to restart markets will be greater. □.

Corollaries 1 and 2 together show that if banks are uncertainty averse, then precise information on $Y_M$ helps to reduce fragility ex-ante, and if collapses occur, to reduce the costs of restarting interbank markets ex-post.

The proposition and corollaries both have the qualifier “all else equal” because individual banks may respond to a coarsening of information on $Y_M$ by improving the quality of their own information disclosures, which could reduce the size of the set $C(\omega, \overline{\omega})$. Several institutions did in fact belatedly improve the quality of their disclosures during the global financial crisis of 2007-2009. The question is whether private incentives are sufficient for resolving these informational problems, or whether there is a role for government in improving disclosures.

As noted in proposition 4, during a crisis, private disclosures on positions may still not be adequate ex-post because of signalling costs. During good times, incentives to disclose information may be even weaker because the uncertainty premia for failing to disclose information can be very low, say on the order of 10 basis points or less, as shown in figure 2. For the same reason, private incentives to collect and reveal information on $Y_M$ may also be inadequate during good times. For example, consider again the two asset example as parameterized for the simulations in section 4.3. Figure 6 plots spreads over the riskfree rate as a function of $Y_M[2]$ the supply of asset 2 held by core banks, when information at date 1 shows its expected returns are high. In a departure from earlier analysis in the paper, spreads are presented not just for the worst case beliefs about banks asset holdings, but for all beliefs about bank’s portfolio’s that are possible conditional on $Y_M[2]$. The resulting relationship is the correspondence $Q$ which maps $Y_M[2]$ into the set of spreads that are possible

---

40 In the example there are 20 banks who each have assets normalized to 1. Because the minimal portfolio weight in asset 2 is 0.4 and the maximal weight is 0.6, the smallest amount of asset 2 that the 20 banks can collectively hold is 8, while the maximum amount is 12.

41 The spreads are those that would prevail in a pooling equilibrium in the anonymous brokered part of the interbank market. As before, I assume that the face value of each banks assets is known, as is their leverage, but banks are uncertain about other banks portfolio weights.
given $Y_M[2]$. The upper blue-dashed and lower solid green boundaries of the correspondence are the spreads that correspond to the most pessimistic and optimistic beliefs about asset exposures, with points in between corresponding to all other beliefs on asset allocations.\textsuperscript{42} To take an extreme example, the figure shows when times are very good, then for all values of $Y_M[2]$ spreads never exceed 1/2 basis point. Since, this is very small, the incentives to gather information on $Y_M[2]$ and then making it known to core banks will be low in good times.

During bad times, following bad news about asset 2, knowledge of $Y_M[2]$, the amount of asset 2 held by all core banks is important. As illustrated in Figure 7, depending on $Y_M[2]$, interbank loan spreads in the anonymous brokered segment of the market can range from a low of 20 basis points to as much as 380 basis points.

Although knowledge of $Y_M[2]$ is valuable during bad times, the extent to which information provision about it is favored by banks depends on whether banks are uncertain and uncertainty averse, abbreviated $KU$ hereafter (for Knightian Uncertainty), or whether they understand their environment and behave according the axioms of Savage’s Decision Theory, abbreviated $SDT$ hereafter. It also depends on the amount of noise in $\tilde{Y}_M$. These ideas are discussed in the context of the two asset example when parameterized the same way as for the simulations in section 4.3.

**Policy When Banks and their Financiers are Uncertainty Averse**

When banks are uncertainty averse, then during bad times they will favor finer information on their aggregate exposures because as corollary 1 shows, finer information will lower interbank spreads and increase the total surplus collected from their short-term lending business.\textsuperscript{43}

Whether banks also favor government bank inspections and sequential release of information on bank health depends on $\tilde{Y}_M$ and whether interbank lending breaks down. If interbank lending breaks down, then in the model all banks benefit from sequential government inspections since it breaks a bad pooling equilibrium and helps to restart lending. If $\tilde{Y}_M = 14$ then all banks would also favor or at least be indifferent to sequential government inspections since the inspections would not raise any bank’s spreads, and would lower the spreads of some banks.\textsuperscript{44} If instead $\tilde{Y}_M = 10$, the spread in the ABM will be 200 basis points, and banks whose PD justified a spread of less than 200 basis points would prefer government sequential inspections, but some banks with greater PDs may oppose inspections because if inspected, they would no longer be able to pool in the anonymous brokered market and their own spread would increase. The precise details on how high a bank’s PD must be to oppose sequential inspections is subtle.\textsuperscript{45}

\textsuperscript{42}The best beliefs are found by choosing allocations to minimize $\hat{P}\hat{D}_i$. The minimum is attained when one bank has a high risk portfolio and is lending in the market, and the other banks all hold the same diversified portfolio.

\textsuperscript{43}The surplus increases because uncertainty is reduced.

\textsuperscript{44}Given $C(\omega, \tilde{x})$ in the example, no single bank could have a spread greater than 380 basis points.

\textsuperscript{45}If bank $i$ has a PD that justifies a spread above 200 bps, and the government does not inspect it, it
In sum, when there is uncertainty and banks are uncertainty averse, all banks would benefit from the release of information on the aggregated risk exposures of all core banks, but depending on $\tilde{Y}_M$, some banks may prefer not to have sequential bank inspections. This means there is a tradeoff in policies. If $\tilde{Y}_M$ is very high because information on core and non-core banks are reported together, then a policy of sequential bank exams is more likely to be favored by banks than it would be information $Y_M$ were provided for core banks alone.

**Policy When Banks and their Financiers satisfy Savage’s Decision Theory**

Suppose banks and their financiers have well formed beliefs that take the form of a unique common posterior distribution over $Y_M$ and the portfolio allocations among the core banks based on their knowledge of $\tilde{Y}_M$. This implies they have a well formed posterior over all pairs $(s, Y_M[2]) \in Q$, where $s$ represents the interbank spread in the top tier of the interbank market and where $Q$ is the set of possible interbank spreads that could result if $Y_M[2]$ was known.

Given the posterior distribution the equilibrium spread, $Spread_{SDT}$ is just the conditional expected spread given $Y_M[2]$. Because $Spread_{KU}$ is based on worst case beliefs, $Spread_{SDT} \leq Spread_{KU}$. Because only banks whose individual spreads are less than the equilibrium spread benefit from inspections, it follows that fewer banks would benefit from sequential government inspections if they are SDT maximizers (the SDT setting) than if there is Knightian Uncertainty and banks are uncertainty averse (the KU setting).

There may also be less support for reducing the noise in $\tilde{Y}_M$, at least ex-post after banks know their probabilities of default at date 1. To see why, recall that when banks are uncertainty averse, a noisier signal about $Y_M$ removes constraints on pessimistic beliefs, which unambiguously increases interbank spreads when there is uncertainty aversion. However, when banks use Bayesian inference, if they receive a noisy signal which suggests that all banks are likely to hold a lot of safe assets, they may increase their assessment of the likelihood that core banks hold a lot of safe assets, thus the noise may benefit some core banks ex-post. From an ex-ante perspective, if some core banks have small exposures to safe assets, they may benefit from the noise in an ex-ante perspective by reducing the interbank spread for core banks below the spread they would pay if their true risk was known.

In summary, in a SDT setting there will be less support for government transparency settings than in a KU setting.

**Interpretation**

The analysis above highlights one of the dilemmas faced by policy makers in setting optimal transparency and disclosure policies. In particular, during a crisis if the economic environment is one in which banks (or their financiers) behave in an uncertainty averse fashion when setting spreads and extending credit, then steps that improve transparency are benefits from inspections if it is not inspected since the inspections would eventually lower spreads in the top tier of the interbank market. But if bank $i$ is inspected, it would face higher spreads. It would therefore trade off these considerations in deciding whether it favored inspections.
more likely to improve the functioning of markets and calm the crisis. Conversely, if bank’s behavior is not characterized by uncertainty aversion, then weak banks may benefit from the lack of transparency while strong banks are hurt by it. If policy makers are especially concerned about the weak banks during a crisis, they may be hesitant to release public information during a crisis because it may make the crisis worse for banks that are weak.

One way to judge whether the KU or SDT views of bank and financier behavior during the crisis is more descriptive is to examine banks responses to transparency type policy interventions since they are more likely to be favored in a KU environment. The recent experience with bank stress tests is suggestive of the KU view. Those banks that have been stress-tested have had an easier time raising capital, and several banks that were not initially stress tested made requests to be stress-tested. This experience suggests that serious consideration should be given to transparency type initiatives that improve transparency about core banks before a crisis, as well as policies that inspects banks and reveal information during a crisis.

The analysis in this paper points towards two potential types of transparency policies pre-crisis. The first is policies that improve knowledge of individual core banks risk exposures ex ante. In the paper uncertainty about individual banks risk exposures is modeled via the condition that each bank believes other banks risk exposures lie within a set: \( \omega \in C[\omega, \bar{\omega}] \), but the paper does not specify precisely where the set comes from (although it is motivated by near optimal behavior). Part of enhanced disclosure could be release of information on risk limits on exposures to different types of assets. Enhanced risk limit information refines the size of the set \( C[\omega, \bar{\omega}] \), but need not reveal a banks precise risk exposures. In addition, enhanced disclosure would reveal the size of \( Y_M \) among core banks. These disclosures together would enhance banks ability to form more precise worst case beliefs, which would have the effect of reducing uncertainty premia.

The uncertainty view of policy has two additional implications for policy. First, it suggests the optimal sequencing of policy steps to address a crisis. More specifically, in what order should capital capital injections, loan guarantees, and information provision via a stress test (such as SCAP) be used during a crisis? If banks and other financiers are uncertainty averse, and information provision takes time, the optimal sequencing should first use temporary interbank loan guarantees to prevent markets from totally collapsing during the crisis. If equity injections to reduce spreads follow next, then if spreads are based on worst-case beliefs, and banks are perceived to be in terrible shape, massive injections of capital would be required to have the desired effect on spreads. Returning to the two asset example, without transparency initiatives, if \( Y_M[2] \) is perceived to be 14 when it is 10, to reduce interbank spread to 40 basis points requires 82 percent capital injections for all core banks. If instead before the capital injections, information was provided that \( Y_M[2] = 10 \), this action alone would reduce the required injection to 60 percent. If in addition, the government inspected all banks before capital was injected, then the injections could be better tailored to individual bank needs, which in some cases could reduce the total amount of equity capital that needed to be injected by 50 percent.\(^{46}\)

\(^{46}\)For example, when \( Y_M = 10 \) worst case asset holdings for interbank spreads would imply that 10 banks
crisis should be loan guarantees, information collection and dissemination, and then capital injections.

An additional implication is that making information on $Y_M[2]$ more precise can help restore the functioning of multiple interbank markets. To see how, suppose that banks that participate in the interbank market for Euro-reserves and banks that participate in the Fed Funds market both hold some of the same long-term assets and that one class of assets is impaired. Suppose, for simplicity, that both sets of interbank markets have similar structures, and that the only information available is on the aggregate holding of banks in the core of the Fed funds market. If the information is imprecise, then in the US worries that US banks exposures may be high to the class of impaired assets can cause the US interbank market to collapse. For the same reason, in Europe, worries that European exposures are high (which implies US are low) could cause the European interbank market to collapse. In fact, both the US and European markets could collapse from uncertainty even though the worst case asset holdings that are envisioned in each market may be jointly impossible.\footnote{Note: these worst case beliefs in each market are not internally consistent when held by market participants who participate in the European interbank market, or the US market, but not both. For participants in both markets, internal consistency would require solving for beliefs that satisfy adding up constraints for counterparty credit risk in both markets simultaneously.} In such circumstances, it is straightforward to construct examples where the provision of more precise information on aggregate asset holdings in one market could help restart one or both markets.

In all of the above analysis, banks are only linked to each other by holding similar assets, but not through any other linkages. Yet, how the banks are linked to each other, which I refer to as the financial architecture, is important. This is briefly discussed in the next section.

5 Financial Architecture

Proposition 4 showed that there are circumstances when government provision of information on bank’s health can restart the interbank market and improve welfare. Critical assumptions for the government’s policy to be successful are that when the government inspects a bank, it can easily infer its solvency; and when it releases information about other banks, the release of that information allows others to improve their inferences concerning the default probabilities of the remaining banks in the system. The extent to which the government’s policy is successful depends upon the architecture of the banks in the financial system, and how they may be linked. In particular, there are settings where knowledge of a banks

\footnote{Note: these worst case beliefs in each market are not internally consistent when held by market participants who participate in the European interbank market, or the US market, but not both. For participants in both markets, internal consistency would require solving for beliefs that satisfy adding up constraints for counterparty credit risk in both markets simultaneously.}
own risk exposures are not sufficient to infer its solvency. This suggests that there may be situations where release of information on individual banks health may not be sufficient to restart the markets. A stark example of this possibility, based on Allen and Gale (2000), is the following: Consider four banks, A,B,C,D. The balance sheets of bank A is as follows:

<table>
<thead>
<tr>
<th>Table 1: Balance Sheet of Bank A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
</tr>
<tr>
<td>Loans 100</td>
</tr>
<tr>
<td>Liabilities</td>
</tr>
<tr>
<td>Deposits 80</td>
</tr>
<tr>
<td>Equity 20</td>
</tr>
</tbody>
</table>

Off balance sheet: Sold “bullet” protection on bank D that pays 25 if bank D defaults.

Banks B,C, and D, have identical balance sheets to bank A, except that bank B has sold protection on A that pays 25 if A defaults. Similarly C has sold protection on B, and D has sold protection on C.

Examination of Bank A in isolation shows that A is solvent if D is, but not if D is not. Therefore, examination of bank A alone does not establish whether A is solvent. Examination of D shows D is solvent if C is solvent, but not otherwise. This means that one cannot know if A is solvent unless one knows whether C is solvent. Therefore knowledge of the exposures of bank A, and of the exposures of the bank that A is directly exposed to does not provide information that is sufficient to know whether A is solvent. Because the example is symmetric, what is true for bank A is true for the other banks. No bank in this example can know its own solvency based solely on its own direct risk exposures.

To examine whether A is solvent, pretend that all of the risk exposures among the banks are known. With this knowledge is A solvent? One definition of solvency is if A defaults, then can A pay off all of its obligations. By this definition, A is certainly insolvent. To see how, note that if A defaults, then B will become insolvent, which will cause C and then D to become insolvent. Finally, when D is insolvent, A will not be able to pay its obligations, so A is definitely insolvent by this definition.

Now, suppose instead that bank C has 30 in equity, instead of 20, and has as an additional 10 in US treasuries on the other side of its balance sheet. With these asset holdings, if A defaults, then so will B, but C and D will not. Hence, A will be able to pay off its liabilities and hence will not be insolvent. Put differently, A will not have a reason to default in the first place.

Now, suppose instead we start the chain with C, then if C defaults, D defaults, which causes A and then B to default. However, when B defaults, if we tally up C’s losses, then C has more than enough assets to pay its obligations. So, it is not consistent to start a chain with C defaulting.

This reasoning suggests that depending on the financial architecture and firms positions:
1. There may be circumstances where no bank knows its own solvency based on its knowledge of its direct risk exposures to other banks.

2. Private parties may not have sufficient information to evaluate the risk of their individual counterparties even if they know that counterparties exposures.

3. The government’s approach to restarting markets in the previous section may fail because the government’s provision of information on asset positions could conceivably not provide sufficient information on the health of individual banks to establish whether they are solvent. In particular, in the network above, if the government made public bank A’s balance sheet, and its connections to banks B and D, that information would not be sufficient to establish whether bank A is solvent.

These results suggests the importance of a comprehensive approach for improving markets; that is, a stress-test type policy of audits and information release is likely to work better if cross-linkages among banks due to counterparty risk are well understood and controlled. If policy initiatives such as central clearing help to control interlinkages due to counterparty risk, then they may also improve the usefulness of stress tests based on risk exposures. How well this can work in practice remains a topic for future research.

6 Conclusion

When one bank makes a loan to another, the credit risk of the bank that lends funds is related to the risk of the assets on the borrowing banks balance sheet, but because of opaqueness, the creditor bank will be uncertain about the borrowing banks portfolio composition, and thus uncertain about the risks that it faces. Nevertheless, despite the uncertainty, unsecured overnight interbank borrowing and lending is common — and spreads for interbank loans at the top tier of the Fed Funds market is typically low. In this paper, I show that inter-bank spreads should contain an uncertainty premium related to credit risk, but because of institutional features of the Federal Funds interbank market, the spread can usually be kept low. Nevertheless, market collapses due to uncertainty are possible. In those circumstances, private efforts to restart markets may not be successful — but sometimes government provision of information as advocated in this paper, or the Bank Holiday of 1933 or the recent Stress Capital Tests that were conducted on US banks can help alleviate uncertainty, restart markets, and improve welfare. Our analysis shows that there is scope for governments to step in ex-post to reduce uncertainty, and furthermore that policies which release better aggregate information on core banks that are at the center of the financial system can help reduce uncertainty ex-ante. An advantage of policies that release aggregate information is that it keeps banks individual exposures private, while still providing information that may improve transparency and financial stability. The success of transparency efforts also depends on how banks are linked to each other in the financial system. How to account for these linkages while formulating a transparency policy remains a topic for future research.
Appendix

A Derivation and Proof of Proposition 1

This section of the appendix provides a formal derivation of the return distribution for the loans in each sector of the economy. Section C of the appendix studies whether the modeling technique that is used to derive a gaussian distribution for portfolios of loans can also generate reasonable default probabilities.

In each sector $m$ there is a continuum of infinitesimal potential long-term borrowers indexed by $\eta_m \in [0, \bar{\eta}_m]$. In this expression, $\eta_m$ denotes borrower $\eta$ in sector $m$, and $\bar{\eta}_m$ is the measure of potential borrowers in sector $m$. The sectors may vary in size — so that for some sectors, such as housing, $\bar{\eta}_m$ is very large, while for other sectors, $\bar{\eta}_m$ is small. The distinction in sector sizes will not be important for the analysis in the paper until section 4.4. Each borrower $\eta_m$ requires 1 dollar of bank financing for a project that returns $r_{\eta_m}$ in period 2, where:

$$r_{\eta_m} = \theta_m + \gamma_m F + \epsilon_m + u_{\eta_m}.$$  \hfill (A1)

The return depends on a sector-specific constant, $\theta_m$, and three independently distributed components. $F$ is normally distributed K-vector that represents news about the macro-economy. $\epsilon_m \sim \mathcal{N}[0, \sigma^2(\epsilon_m)]$ represents news about sector $m$, and is distributed independently of news in sectors $m' \neq m$. Finally, $u_{\eta_m}$ is a borrower-specific component that is independent across borrowers and distributed uniformly on $[0, \bar{u}_m]$.

The macroeconomic factor can be further decomposed into components $f(1)$ and $f(2)$:

$$F = f(1) + f(2),$$  \hfill (A2)

where $f(1)$ is the best forecast of $F$ conditional on public information $I_1$ that arrives at date 1 ($f(1) = E[F|I_1]$), and $f(2)$ represents the error in the forecast which is learned at date 2.

Because $f(1)$ and $f(2)$ are innovations in beliefs about $F$, they have mean 0, and are uncorrelated; for tractability they are assumed to be normally distributed:

$$\begin{pmatrix} f(1) \\ f(2) \end{pmatrix} \sim \mathcal{N} \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_{f(1)} & 0 \\ 0 & \Sigma_{f(2)} \end{pmatrix} \right).$$  \hfill (A3)

Each loan that is extended to a borrower in sector $m$ promises to pay back contractually agreed upon principal plus interest $X_m$ at maturity, but in the case of default only produces the non-stochastic recovery value $RGD_m$.\footnote{$RGD_m$ stands for the recovery given default in sector $m$. It is not restricted to depend on $X_m$.} The loan defaults if the rate of return on the
entrepreneurs project is less than $X_m$. For the purposes of this paper, $X_m$ is a fixed parameter that is determined by the financial intermediation process for long-term loans. I abstract from its determination because it is not essential for the analysis of the interbank market.

We will make the following additional approximating assumption about the distributions:

**Assumption 5** For all values of $F$ and $\epsilon_m$,

$$0 < X_m - (\theta_m + \gamma_m F + \epsilon_m) < \bar{u}_m$$

To a first approximation, the assumption can be understood as requiring that the variability in borrowers returns due to the macro factors $F$ and sector specific risk $\epsilon_m$ are small relative to the variability due to the borrowers idiosyncratic risk since $[0, \bar{u}_m]$ represent upper and lower bounds of the idiosyncratic return risk. This assumption cannot literally be true because $F$ and $\epsilon_m$ are Gaussian random variables, but in appendix C we show that the returns on loans can be calibrated so that the probability the assumption is violated is approximately $10^{-7}$. For simplicity and tractability we assume that it is true. Under the condition that it is true,

$$\text{Prob}(\eta_m X_m I_0 \sim N[\mu, \Sigma]) \quad (A5)$$

$$\text{Prob}(R_m \mid I_1 \sim N[\mu(1), \Sigma(1)]) \quad (A6)$$

From this probability, conditional on $F$ and $\epsilon_m$ it then follows by the law of large numbers that for any positive mass of loans in sector $m$, the fraction that default in period 2 is given by the expression on the right hand side of equation A4. Because $F$ and $\epsilon_m$ are normally distributed conditional on the information sets $I_0$ and $I_1$, it then follows that the fraction of loans from sector $m$ that will default in period 2 is also normally distributed conditional on $I_0$ and $I_1$.

Let a sector portfolio denote a portfolio of loans to a positive mass of borrowers in sector $m$. Because $X_m$ and the recovery given default are non-stochastic, the return per dollar to the sector-portfolio, denoted $R_m$, is proportional to the fraction of borrowers that default, and is also normally distributed. Additionally, the vector of returns per dollar for loans in each sector, denoted by $R = (R_1, \ldots R_M)'$, is jointly normally distributed. This result, and the details of the conditional distribution functions is stated formally below:

**Proposition 1:** Under assumption 5, $R$, the gross return on an $M$-vector of sector portfolios has the following distribution conditional on $I_0$ and $I_1$:

$$R \mid I_0 \sim N[\mu, \Sigma]$$

$$R \mid I_1 \sim N[\mu(1), \Sigma(1)]$$

39
where,

\[
\mu = \alpha,
\]

\[
\mu(1) = \alpha + \beta f(1),
\]

\[
\alpha = \begin{pmatrix}
\alpha_1 \\
\vdots \\
\alpha_M
\end{pmatrix}, \quad \beta = \begin{pmatrix}
\beta'_1 \\
\vdots \\
\beta'_M
\end{pmatrix}
\]

\[
\alpha_m = \left[ X_m + (RGD_m - X_m) \left( \frac{X_m - \theta_m}{\bar{u}_m} \right) \right],
\]

\[
\beta_m = \left[ \frac{(X_m - RGD_m) \gamma_m}{\bar{u}_m} \right],
\]

\[
\Sigma = \beta [\Sigma_f(1) + \Sigma_f(2)] \beta' + \begin{bmatrix}
\sigma^2(\epsilon_1) & \cdots \\
\vdots & \ddots \end{bmatrix},
\]

\[
\Sigma[1] = \beta [\Sigma_f(2)] \beta' + \begin{bmatrix}
\sigma^2(\epsilon_1) & \cdots \\
\vdots & \sigma^2(\epsilon_M)
\end{bmatrix}.
\]

**Proof:** See the derivation above the proposition. \( \square \).

The proposition is a straightforward application of the law of large numbers and closely follows Vasicek’s(2002) development of asymptotic portfolios. However, in Vasicek, the returns on asymptotic portfolios are a nonlinear function of the factors, which is unwieldy for theoretical modeling with even a single factor. By contrast, the derivation here presents a linear result, which is useful for many theoretical modeling applications, including the ones considered here.

### B Other Propositions and Proofs

**Proposition 2:** Although there is Knightian uncertainty about bank i’s positions, under some economic conditions (such as case 1), favorable economic news can reduce the loan spread that bank i pays for borrowing in the interbank market even though there is Knightian uncertainty over bank i’s positions. Conversely, under other economic conditions, such as those in case 2, sufficiently unfavorable news about some sectors of the market can destroy bank i’s ability to finance its new lending opportunities in the interbank market, effectively causing the interbank market to break down.

**Proof:** Assume \( \omega_i \in C(\omega_i^*) \) maximizes \( PD_i(\omega, 0) \) the probability that i defaults conditional on information at time 0, and \( \bar{\omega}_i \) maximizes \( PD_i(\omega, 1) \), the probability that i defaults.
conditional on the information at time 1. \( PD_i(\omega_i, 0) > PD_i(\tilde{\omega}_i, 1) \) if:

\[
\frac{L_i}{1+L_i} R^D - \omega_i' \alpha > \frac{L_i}{1+L_i} R^D - \tilde{\omega}_i' \alpha
\]

Adding and subtracting \( \frac{L_i}{1+L_i} R^D - \tilde{\omega}_i' \alpha \) to the right hand side and rearranging shows the inequality holds if:

\[
\frac{L_i}{1+L_i} R^D - \omega_i'(\alpha + \beta f(1)) \frac{1}{\sqrt{\omega_i' \Sigma \omega_i}} > \frac{L_i}{1+L_i} R^D - \tilde{\omega}_i' \alpha \frac{1}{\sqrt{\omega_i' \Sigma \omega_i}}
\]

- \( \tilde{\omega}_i' \beta f(1) \frac{1}{\sqrt{\omega_i' \rho \Sigma \omega_i}}
\]

The left hand side of the inequality is positive because \( \omega_i \) maximizes the default probability at time 0. By the assumptions of the proposition, \( \left( \frac{L_i}{1+L_i} R^D - \tilde{\omega}_i' \alpha \right) \) is negative, and \( \rho < 1 \). This guarantees that the first expression on the right hand side is negative. Since \( \omega_i > 0 \) and \( \beta f(1) > 0 \), the second term on the right hand side is also negative. This establishes the inequality is true, and shows that under some conditions the news at time 1 is unambiguously good.

To show that in the second case the news can be unambiguously bad, note that the risk exposure to asset 1 can be positive and the news about the mean return of asset 1 is negative. It then follows that if the mean on asset 1 is sufficiently low, and the possible exposure is sufficiently high, then the interbank market will freeze up, as claimed in the proposition. \( \square \)

**Proof of Proposition 3**

To prove proposition 3, I first solve for the worst case default beliefs that any borrowing bank believes any lending bank could have. The formal solution for the worst case beliefs is slightly complicated because it involves solving for the portfolio of the lending bank and the borrowing banks, and ensuring that the borrowing banks have the highest probability of default possible. This generates four special cases, as illustrated below:

**Lemma 1** Under assumption 4, the worst case beliefs \( \hat{PD}_i \) that solve 16 are given by the following:

If \( PD(\overline{\omega}) \geq PD(\underline{\omega}) \), then for \( m_u \) and \( \tilde{\omega} \) that satisfy the conditions,

\[
\begin{align*}
    m_u &= \max m, \quad m \in \{1, \ldots, 2N\}, \quad \text{such that} \\
    Y_M/A_k &= m\overline{\omega} + (2N - m - 1)\underline{\omega} + \tilde{\omega}, \\
    \tilde{\omega} &\in C(\underline{\omega}, \overline{\omega}),
\end{align*}
\]

41
if $PD(\tilde{\omega}) \leq PD(\omega)$, then
\[
\hat{PD}_i(ABM) = \frac{m_uPD(\overline{\omega}) + (2N - m_u - 1)PD(\omega)}{2N - 1};
\]

if $PD(\tilde{\omega}) \geq PD(\omega)$, then
\[
\hat{PD}_i(ABM) = \frac{m_uPD(\overline{\omega}) + (2N - m_u - 2)PD(\omega) + PD(\tilde{\omega})}{2N - 1}.
\]

If $PD(\overline{\omega}) \leq PD(\omega)$ then for $m_u$ and $\tilde{\omega}$ that satisfy the conditions,
\[
m_u = \max m, \quad m \in \{1, \ldots, 2N\}, \quad \text{such that}
\]
\[
Y_M/A_k = m\omega + (2N - m - 1)\overline{\omega} + \tilde{\omega},
\]
\[
\tilde{\omega} \in C(\omega, \overline{\omega}),
\]

if $PD(\tilde{\omega}) \leq PD(\overline{\omega})$, then
\[
\hat{PD}_i(ABM) = \frac{m_uPD(\omega) + (2N - m_u - 1)PD(\overline{\omega})}{2N - 1};
\]

if $PD(\tilde{\omega}) \geq PD(\overline{\omega})$, then
\[
\hat{PD}_i(ABM) = \frac{m_uPD(\omega) + (2N - m_u - 2)PD(\overline{\omega}) + PD(\tilde{\omega})}{2N - 1}.
\]

**Proof:** For each borrower bank, the portfolio that maximizes its probability of default maximizes the opposite of a Sharpe ratio given by
\[
\frac{L_i}{1 + L_i}R^D - \omega_i\mu(1)
\]
when treating $(R^D L_i/(1 + L_i)$ as the risk-free rate. Since the Sharpe ratio is a concave function of a portfolio weights, minus the Sharpe ratio is a convex function of the weights. The probability of default depends on $\Phi(\cdot)$ of the opposite of the Sharpe ratio. Under assumption 4, the operational part of the $\Phi(\cdot)$ function is its lower half, which is a convex function. Since convex functions of convex functions are convex, the probability that any bank defaults is a convex function of its portfolio weights. Since the objective function being maximized is a positive weighted sum of convex functions, it is also a convex function. Inspection will also quickly reveal that the feasible set of portfolios is a convex set. When a convex function is maximized over a convex set, the solution is at the extremes. This implies that no more than one of the $2N$ banks can have a portfolio weights that are not equal to either $\overline{\omega}$ or $\omega$ since if two banks had portfolios in the interior of the set, they would be able to alter
their portfolios until one of them hit a boundary. Finally, after the optimal portfolio weights are found, the worst case beliefs that a lender could have would involve a lender having the lowest risk possible, and the borrowing banks having the highest risk possible. Applying this criterion generates the four conditions in the final result for $\hat{PD}(ABM)$. □.

Proposition 3: Under assumption 4, and the constraints on beliefs from equations (17) and (18), when there is uncertainty, the spreads that banks pay in the anonymous brokered tier of the interbank market are less than or equal to the spreads they would pay when borrowing in the bilateral tier of the interbank market.

Proof: The result follows directly from the lemma, and the discussion above proposition 3 in the text. □.

Proposition 4 There exist $P_H$, $P_L$, $N$, $M$, $c$, $LGD_{i,j}$ and $R_l$ such that:

1. 1 Because of uncertainty about banks risk exposures the interbank market may break down.

2. 2 Private provision of information on the exposures may be too costly to restart the market.

3. 3 Sequential government-supported inspections in which the government inspects banks, and announces their health and risk exposures may restore market functioning and improve social surplus. This may be possible even if the government faces higher costs of information gathering than the banking sector faces.

4. 4 There are parameter values for which results 1 - 4 hold, and for which the expected social surplus from sequential government-supported inspections can be made arbitrarily large.

Proof: For simplicity assume all banks with a surplus of funds are ex-ante symmetric, and that all banks with a shortage of funds are ex-ante identical, and that all short-term borrowers have the same reservation rate for borrowing given by $\bar{R}_l$. Furthermore, banks assets holdings correspond with those associated with worst case beliefs so that there are $2N - 1$ banks with low default probability $P_L$ and one with high default probability $P_H$.

If bank $i$ makes a short-term loan and does not reveal information about itself first, then the social surplus that it creates will be negative if $P_H$ and $P_L$ satisfy:

$$\text{Surplus}_i = \bar{R}_l - \left( R^D + [P_L + (1/(2N - 1))(P_H - P_L)]\ LGD_i \right) = \eta, \quad (A7)$$

for some small $\eta < 0$.

If the cost of signalling the information to bank $i$ is $c$ given by

$$c = (\bar{R}_l - R^D - P_L\ LGD)(1 + \frac{M - 1}{2}) + \epsilon, \quad (A8)$$
for some small $\epsilon > 0$, then the cost of signalling exceeds the expected benefits that the bank could hope to earn from signalling immediately and making a loan now plus the expected future benefits from having signalled.

To prove results 1 and 2, choose $P_H$ and $P_L$ and $LGD_i$ such that $1/2 > P_H > P_L > 0$, and $1 > LGD_i > 0$. Given these choices, choose $\bar{R}_i$ so that $\bar{R}_i = R^D + [P_L + (1/(2N - 1))(P_H - P_L)]LGD_i + \eta_i$ with $-(1/(2N - 1))(P_H - P_L)LGD_i < \eta_i < 0$. These conditions guarantee that without signaling the spread charged to bank $i$ will be too high, and will cause the interbank market to breakdown, proving result 1. Choosing $c$ as in equation A8 guarantees that bank $i$’s costs from signalling exceed its expected benefits, so it will not signal. The same will be true for all banks $i$, proving result 2.

To prove 3, assume the government performs sequential information collection (i.e. sequential search) in which it looks at each bank, certifies whether it is good, or bad, discloses information about the banks risk exposures, and stops searching after it has found the single bad bank, which is shut down. The expected search costs are equal to $c(2N - 1)/2$. Abstracting from the integer problem, of the $2N - 1$ good banks, each is expected to be a borrower in the interbank market in $1/2$ of the periods $1, \ldots M$; in each period the borrowing bank captures surplus $\bar{R}_i - R^D - P_LLGD_i$, for a total expected surplus net of borrowing costs of:

$$\text{Surplus} = [(2N - 1)/2](\bar{R}_i - R^D - P_LLGD_i)M - [(2N - 1)/2]c$$

$$= [(2N - 1)/2](\bar{R}_i - R^D - P_LLGD_i)M - [(2N - 1)/2][(\bar{R}_i - R^D - P_LLGD)(1 + \frac{M - 1}{2}) + \epsilon]$$

$$= [(2N - 1)/2](\bar{R}_i - R^D - P_LLGD_i)(\frac{M - 1}{2}) - [(2N - 1)/2]\epsilon.$$

The final line of the expression for surplus can be guaranteed to be positive by choosing $\epsilon$ sufficiently small. This establishes result 3 when the government and private sector have the same cost of monitoring. If $c$ for the government is equal to $\psi$ times the private cost of monitoring, for $\psi < 2$, it is straightforward to reevaluate expected surplus and show that it will still be positive, completing the proof of result 3.

Finally, holding $N$ fixed, but allowing $M$ to approach infinity, with the resulting consequences for $c$, shows from the last equation that there are economies in which the interbank market will break down, private efforts to restart it will fail, and for which the expected social surplus from the governments efforts to restart the market are arbitrarily large. This establishes result 4. $\square$.

**Proposition 5:** If there are 2 or more small banks and two or more large banks that want to borrow in the anonymous brokered tier of the interbank market, then if $\sum_{s=1}^{S} A_s << Y_M[1]$, the robust probability of default for the two asset example in equation 21 is increasing in the percentage of small banks that participate in the anonymous brokered tier of the market.

\footnote{At most $2N - 1$ searches are needed, and the chance of finding the high risk bank on each search attempt without replacement is $\frac{1}{2N}$. Elementary calculations then show the expected search cost is as given.}
**Proof:** The robust probability of default is increasing if and only if

\[
\frac{P'_H + (2N - 2)P_L + SP_S}{2N - 1 + S} > P_L + \frac{1}{2N - 1}(P_H - P_L).
\]

Since \(P'_H > P_L\), the inequality will be satisfied if

\[
\frac{P_L + (2N - 2)P_L + SP_S}{2N - 1 + S} > P_L + \frac{1}{2N - 1}(P_H - P_L),
\]

which reduces to the condition

\[
\frac{P_S - P_L}{P_H - P_L} > \frac{1}{2N - 1} + \frac{1}{S}.
\]

Because \(P_S > P_H\) the left hand side is greater than 1, while since \(N\) and \(S\) are greater than two, the right hand side is less than 1. \(\square\)

**C Adequacy of the normality approximation**

This appendix illustrates that the Gaussian approximation for the returns of loan portfolios is reasonable.

Recall that when the returns for infinitesimal investors in sector \(m\) satisfy equation A1:

\[
r_{s,m} = \theta_m + \gamma_m F + \epsilon_m + u_{s,m}.
\]

where \(F\) and \(\epsilon_m\) are independent and gaussian, and \(u_{s,m} \sim \text{Uniform}[0, \bar{u}_m]\).

then for \(F\) and \(\epsilon_m\) that satisfy the regularity condition 5 :

\[
X_m - \theta_m > \gamma_m F + \epsilon_m > X_m - \theta_m - \bar{u}_m,
\]

the probability that a loan made at rate \(X_m\) to entrepreneur \(s\) in sector \(m\), will default conditional on \(F, \epsilon_m\) is

\[
\text{Prob}(r_{s,m} < X_m|F, \epsilon_m) = \frac{X_m - \theta_m - \gamma_m F - \epsilon_m}{\bar{u}_m}.
\]

This implies that in a well diversified portfolio, conditional on the information provided at time 0, the expected proportion of defaults, which is the unconditional probability of default is \(\frac{X_m - \theta_m}{\bar{u}_m}\). If the rates on the loans are set competitively at time 0, then \(X_m\) should be set so that investors are indifferent between extending this loan or holding the riskfree asset. This implies that \(X_m\) solves the quadratic equation:

\[
X_m \left[ 1 - \frac{X_m - \theta_m}{\bar{u}_m} \right] + \left[ \frac{X_m - \theta_m}{\bar{u}_m} \right] RGD = R_f,
\]

45
where \( R_f \) is the gross riskfree rate and \( X_m \) is the gross interest rate in the sector.

To illustrate whether our approach can be used to generate reasonable default probabilities for one sector alone we solved the model with one macro factor \( F \) and without a sector specific shock (which is unnecessary to include in the one sector case). The parameters are as follows:

\[
\begin{align*}
\theta_m &= 0.55 \\
\gamma_m &= 1 \\
\sqrt{\sigma_F^2} &= 0.1 \\
\bar{u}_m &= 10 \\
RGD &= 0.5 \\
R_f &= 1.02
\end{align*}
\]

With these parameter choices, assuming regularity condition 5 is satisfied, and that the probability of default can have a Gaussian distribution, the quadratic equation implies \( X_m \) is 1.0472. Given this choice of \( X_m \) the unconditional probability of default, \( \frac{X_m - \theta_m}{\bar{u}_m} \), is .04972. To check the approximate internal consistency of these values, I generated 10 million monte-carlo draws from the true return distribution and estimated the default probability to be the fraction of return draws below 1.0472. A plus or minus two standard deviation confidence interval for the default probability is \([.0495, .0498]\), which contains the approximate probability of default.

To investigate the probability that condition 5 will be violated, note that the probability of a violation is

\[
\begin{align*}
\text{Prob(Violation)} &= 1 - \text{Prob} \left( X_m - \theta_m > \gamma_m F + \epsilon_m > X_m - \theta_m - \bar{u}_m \right) \\
&= 1 - \left[ \Phi \left( \frac{X_m - \theta_m}{\sqrt{\gamma_m^\prime \Sigma_m^2 \gamma_m + \sigma^2(\epsilon_m)}} \right) - \Phi \left( \frac{X_m - \theta_m - \bar{u}_m}{\sqrt{\gamma_m^\prime \Sigma_m^2 \gamma_m + \sigma^2(\epsilon_m)}} \right) \right] \\
&= 1 - \Phi(4.9721) - \Phi(-95.028) \\
&= 3.31 \times 10^{-7}
\end{align*}
\]

The requirement that the fraction of defaults predicted by the model conditional on the factors is between 0 and 1, i.e. that the normality assumption is reasonable reduces to the condition in 5.

The results here illustrate that using a normal distribution as an approximation for default probabilities and the return on a portfolio of loans from different sectors of the economy can be reasonable method for approximating default behavior, and creating returns on sector portfolios that are normally distributed.

There is one drawback of this approach. For simplicity, I used a uniform distribution for the variable \( u_{m,s} \) over a support on \([0, 10]\). If one uses a uniform random variable for the
idiosyncratic return component, then its support has to be very large to generate plausible default probabilities. For example the upper bound 10 corresponds to a gross return of 1000 percent, with a gross average return of around 500 percent. This is not a problem unless one is hoping to also match data on the returns that entrepreneurs actually earn.

To match the entrepreneurs returns, then it is better to model $u_{s,m}$ as a random variable that has constant density for low values and then has a different distribution (such as terminating at a mass point) for higher values. For example, if $\text{Prob}(u_{s,m} < k) = k/10$ for $0 \leq k < 1.5$, and $\text{Prob}(u_{s,m} = 1.5) = 0.85$, then the upper bound on gross returns is much smaller, and will better fit the data, and by the law of large numbers the return for a well diversified portfolio will still turn out to be approximately Gaussian.
BIBLIOGRAPHY


Table 1: Balance Sheet of Bank A

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans 100</td>
<td>Deposits 80</td>
</tr>
<tr>
<td>Equity 20</td>
<td></td>
</tr>
</tbody>
</table>

Off balance sheet: Sold “bullet” protection on bank D that pays 25 if bank D defaults.
Figure 1: **Investment Opportunity Set for a Bank**

*Notes:* For the bank optimization problem in section 3, the Figure illustrates the banks investment opportunity set in terms of the mean and standard deviation of the portfolios it can choose. The bank chooses its portfolio subject to a credit value at risk constraint (points on and above and to the left of the purple line satisfy the constraint). Its optimal portfolio is \( \bar{\omega} \), but because of internal constraints, it is assumed the bank instead chooses some mean-variance efficient portfolio between \( \omega \) and \( \bar{\omega} \).
Notes: This Figure is reproduced from the author, forthcoming (2009). For a stylized bank that holds two risky assets in its loan portfolio, the figure presents surface and contour plots of the uncertainty premium (in basis points) that the bank pays for its short-term unsecured interbank borrowing as a function of its leverage and as a function of the volatility (standard deviation) of its assets relative to its baseline value.
Notes: For a stylized example in which the interbank market breaks down due to uncertainty over banks' risk exposures to a distressed class of assets, the figure presents a graph of the social surplus that is generated when the government follows a policy that involves sequentially inspecting and announcing the health and asset holdings of individual banks. The searches stop when liquidity is restored to the interbank market and when the surplus from interbank loans is judged to be high enough. Social surplus is a function of the true concentration of risk exposures within the banking sector to the distressed class of assets. Social surplus is measured in units of return per dollar of new entrepreneurial loans due to restoration of lending in the interbank market. Concentration is measured by an index that varies from 0 to 1, where 0 represents minimal concentration of risky holdings, and 1 denotes maximal concentration of risky holdings. Additional details are provided in the text.
Figure 4: Economic Surplus from Government’s Sequential Inspection and Announcement Policy with Varying Government Inspection Costs and Random Search

Notes: For the inspect and announce policy in Figure 3, this figure reports the surplus that is created if government’s costs of evaluating each bank are equal to the banking sectors own costs multiplied by factors of 1.0, 1.1, 1.2, 1.3, etc... Results are presented for differing levels of concentration of banking sector exposure to the distressed asset class. For further details see Figure 3.
Figure 5: Economic Surplus from Government’s Sequential Inspection and Announcement Policy with Varying Government Inspection Costs and Search Based on Rank of Exposure

Notes: For the inspect and announce policy in Figure 3, and the cost structure in Figure 4, this figure reports the surplus that is created if government knows the relative magnitude of banks exposures to the distressed asset class and follows an inspect and announcement policy in which banks with higher risk exposures are inspected first, and in which inspections cease after the interbank market restarts.
Figure 6: Interbank Spread as a Function of High Risk Assets Held by Banks: Good Economic Conditions

Notes: For the two risky asset example in section 4.3, for various levels of outstanding supply of risky asset 2, the figure illustrates the spread in a pooling equilibrium that banks have to pay when borrowing in the interbank market when assets are distributed among banks to maximize the spread (blue dashed line) and when assets are distributed among banks to minimize the spread (solid green line). The units on the y-axis are basis points. The figure shows regardless of supply, the spread is very low, i.e. at most it is just over 1/2 basis point.
Figure 7: Interbank Spread as a Function of High Risk Assets Held by Banks: Weak Economic Conditions

Notes: For the two risky asset example in section 4.3, for various levels of outstanding supply of risky asset 2, the figure illustrates the spread in a pooling equilibrium that banks have to pay when borrowing in the interbank market when assets are distributed among banks to maximize the spread (blue dashed line) and when assets are distributed among banks to minimize the spread (solid green line). The units on the y-axis are basis points. The figure shows in weak economic conditions the spread is very sensitive to the known outstanding amount of the risky asset held by banks.