1. Definitions: 2 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.
Whenever possible, offer both a verbal and a mathematical definition.

Def 1. closed set

Def 2. compact set

Def 3. function

Def 4. derivative (of a function)

Def 5. gradient (of a function)

Def 6. Hessian (of a function)

Def 7. differential form (of a function)

Def 8. positive definite quadratic form

Def 9. extremum

Def 10. implicit function theorem

Def 11. concave function

Def 12. envelope theorem

Def 13. characteristic roots (of a square matrix)

Def 14. binary operation

Def 15. leading principal minor (of a square matrix)

2. Multiple Choice: 3 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED. (Please circle your answers.)

MC1. Which of the following sets are equal to $\{2, 4, 6, 8\}$?

(a) $\{2, 2, 4, 4, 6, 6, 8, 8\}$

(b) $\{x \in \mathbb{Z}_{++} | 0 < x/2 < 5\}$

(c) $\{x \in \mathbb{Z}_{++} | 0 < x < 10\}$

(d) a. and b.

(e) all of the above

MC2. Suppose you wish to graph a function f over the interval $[0, 3]$, and you decide to use the endpoints of the interval along with an evenly spaced sample of points in between? Which of the following GAUSS statements might represent this decision?

(a) `seqa(0,0.1,30)`

(b) `seqa(0,0.1,31)`

(c) `seqm(0,0.1,30)`

(d) `seqm(0,0.1,31)`

(e) none of the above

MC3. Given a vector $\mathbf{x}$ of real numbers, what is returned by the GAUSS command `selif(X,X%3 .eq 0)`?

(a) The integer approximation of $3\mathbf{x}$.

(b) The elements of $\mathbf{x}$ containing 3 as a digit.

(c) The elements of $\mathbf{x}$ that are evenly divisible by 3.

(d) The first three elements of $\mathbf{x}$ that equal zero.

(e) none of the above

MC4. If a finite sequence has n elements, how many permutations does it have?

(a) n

(b) $n!$

(c) $2n$

(d) 2^n

(e) none of the above

MC5. If the set P contains every element in the set Q, which of the following statements are true?

(a) $P = Q$

(b) $P \supseteq Q$

(c) $|P| = |Q|$

(d) a. and b.

(e) all of the above

MC6. Suppose $P = \{\omega_1, \omega_2\}$. Which of the following is a subset of P?

(a) $\{\omega_1, \omega_2\}$

(b) $\{\omega_1\}$

(c) $\emptyset$

(d) a. and b.

(e) all of the above

MC7. If $\log_b x = \log_b(yz)$, then

(a) $\log_b x = log_b y + log_b z$

(b) the growth rate of x equals the growth rate of y plus the growth rate of z.

(c) $b^x = b^{yz}$

(d) a. and b.

(e) all of the above

MC8. Suppose $0 < b < 1$. Then if we graph b^x we will find

(a) $b^x > 0$ always.

(b) the function has a positive slope.

(c) the function has an increasing slope.

(d) a. and c.

(e) all of the above

MC9. Which of the following describes a hyperbola?

(a) $xy = 1$

(b) $x^2 + y^2 = 1$

(c) $x = y$

(d) $\sin^2 x + \cos^2 x$

(e) c. and d.

MC10. Which of the following describes a circle?

(a) $xy = 1$

(b) $x^2 + y^2 = 1$

(c) $\sin^2 x + \cos^2 x$

(d) $x = y$

(e) c. and d.

MC11. Which of the following describes a straight line?

(a) $xy = 1$

(b) $x^2 + y^2 = 1$

(c) $x = y$

(d) $\sin^2 x + \cos^2 x$

(e) c. and d.

MC12. How do the graphs of $y = x^2$ and $y = (x + 10)^2$ differ?

(a) the second graph is shifted to the left

(b) the second graph is generally "steeper," in the sense that it is stretched vertically.

(c) the second graph is generally "wider," in the sense that it is stretched horizontally

(d) a. and b.

(e) all of the above

MC13. Consider the Cobb-Douglas production function $Y = AL^{0.6}K^{0.4}$ Suppose the continuously compounded annual growth rates of productivity (A), labor inputs (L), and capital inputs (K) are 1%, 3%, and 2%. Then the continuously compounded annual growth rate of output is

(a) less than 1%

(b) 1%

(c) 3.6%

(d) 7%

(e) none of the above

MC14. Suppose your productivity at a particular job increases with experience, so that your effective labor input is $L(t) = 2 - e^{-0.1t}$. Your starting productivity and maximum possible productivity are

(a) $2, \infty$

(b) $1, \infty$

(c) $1, 2$

(d) $-\infty, 2$

(e) $-\infty, \infty$

MC15. Which of the following are properties of the matrix inverse?

(a) $(A^{-1})^{-1} = A$

(b) $(A^{-1})^{\top} = (A^{\top})^{-1}$

(c) $A^{-1}B^{-1} = (BA)^{-1}$

(d) a. and b.

(e) all of the above

MC16. Construct a matrix `A[3,3]={1 2 3 0 1 2 0 0 1 }`. The determinate of this matrix is

(a) 0

(b) 1

(c) 2

(d) 3

(e) 12

MC17. If A is a real 2×2 matrix and $|A| < 0$ then we know the following about the characteristic roots

(a) there are two characteristic roots

(b) the characteristic roots are real numbers

(c) one characteristic root is negative

(d) a. and b.

(e) all of the above

MC18. If $f(x) = \log_2 x$ then $Df(x) =$

(a) $1/2x$

(b) $1/x$

(c) $\ln x / \ln 2$

(d) $1/x \ln 2$

(e) none of the above

MC19. If $f(x) = 10^x$ the $Df(x) =$

(a) 10^x

(b) $10 \cdot 10^x$

(c) $\ln 10 \cdot 10^x$

(d) $1/x \ln 10$

(e) none of the above

MC20. If $h(x) = f(g(x))$ where $g(x) = (x+1)^2$ and $f(y) = y^2$ then $h'(x) =$

(a) $f'((x+1)^2) \cdot g'(x)$

(b) $f'(g(x)) \cdot g'(x)$

(c) $f'(x) \cdot g'(x)$

(d) a. and b.

(e) all of the above

3. Short Answer: 10 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

SA1. For the following functions, find the critical points and use a derivative test to determine whether these are maxima or minima.

- $f(x) = (x-1)^2$
- $f(x) = (x-2)^3$
- $f(x) = (x-3)^4$
- $f(x) = 10 + 5x - x^2$
- $f(x) = 3 - 4x + x^2$

SA2. Consider the general form of a quadratic polynomial: $f(x) = a_0 + a_1 x + a_2 x^2$, with $a_2 \neq 0$. Find the critical point. Is a quadratic polynomial concave or convex? (Give an answer based on a derivative test.) Is the critical point a local or global extremum? Explain.

SA3. Consider the production function $f(K, L, H) = K^{\alpha} L^{\beta} H^{1-\alpha-\beta}$. Find the gradient

SA4. Let $X = \{\text{Alice, Bert, Claire}\}$, $Y = \{\text{algebra,calculus}\}$. What is $X \otimes Y$? Let $R = \{(A, a), (B, a), (C, a), (C, c)\}$. Is R a binary relation on X? (Explain.) Is R a relation from X to Y? (Explain.) Is R a function from X to Y? (Explain.) What is R^{-1}? Construct a matrix that can represent R. Can it also represent R^{-1}?

SA5. Given the three matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \qquad C = \begin{bmatrix} 9 & 10 \\ 11 & 12 \end{bmatrix}$$

Calculate the element in the first row and second column of ABC. Be sure to explain *precisely* what you are doing.

SA6. A simple "Classical" IS-LM model can be written as

$$Y = g - \beta R \qquad\qquad m = \kappa Y - \gamma R$$

where the endogenous variables ("unknowns") are R and m and the exogenous variables ("constants") are g and Y. Write the model as a matrix equation in the form $Ax = b$ and solve for the reduced form using matrix algebra.

SA7. A simple "Classical" IS-LM model can be written as

$$Y = ad(R, g) \qquad\qquad m = L(R, Y)$$

where the endogenous variables ("unknowns") are R and m and the exogenous variables ("constants") are g and Y. Write the total differential of this model as a matrix equation in the form $Ax = b$ and solve for the reduced form partial derivatives using matrix algebra.

SA8. Consider the following GAUSS procedure:

```
proc (1)=bisect(&f,rt1,rt2);
  local eps,mid,f:fn;
  eps=1e-9;
  if f(rt1)*f(rt2)>0;stop;endif;
  do while abs(rt2-rt1)>eps;
    mid=(rt1+rt2)/2;
    if f(rt1)*f(mid)>0;rt1=mid;
      else; rt2=mid;
    endif;
  endo;
retp((rt1+rt2)/2);
endp;
```

Give a general verbal discussion of the algorithm ("bisection method") that this procedure implements. Be sure to provide a graphical illustration. Then comment each line of code to explain precisely how the procedure implements the bisection method.

SA9. Suppose your utility function is $U(c_1, c_2) = c_1^{1/2} c_2^{1/3}$ and you face prices $p = (1/2, 1/3)^\top$. Given wealth of \$10,000, what is your optimum consumption bundle? Set up and solve as an unconstrained optimization problem.