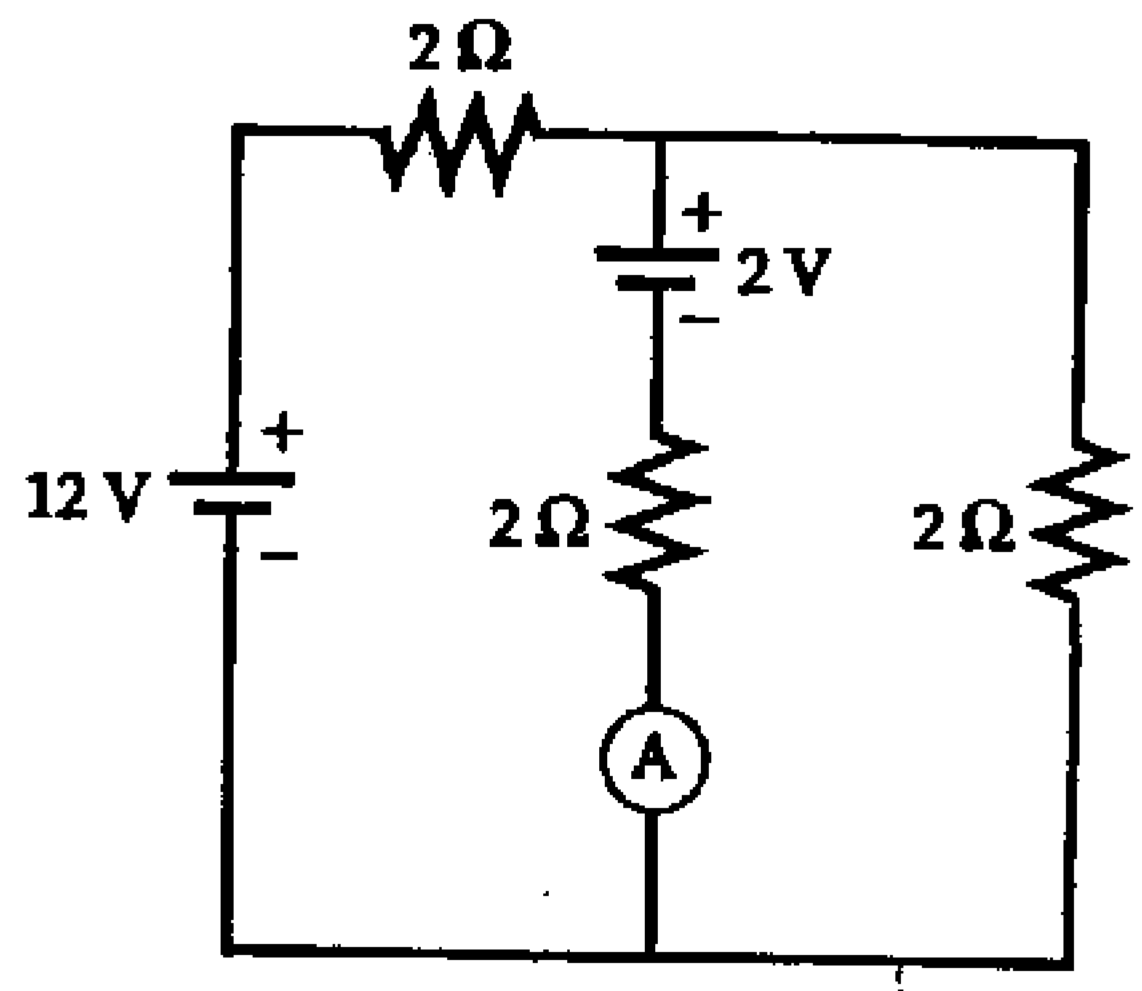
12 I. For the circuit to the right, determine the following. (The ammeter has negligible resistance.)

4 (a) The current through each resistor. Be sure to indicate direction as well as magnitude for each current.

2 (b) The power supplied by the 12V battery.

3 (c) The power dissipated in each resistor.

3 (d) Explain the difference in your answers to (b) and (c).



18 II. For the circuit shown below, determine

2 (a) The battery current immediately after the switch S is closed.

2 (b) The battery current a long time after S is closed.

2 (c) The potential difference across the $600 k\Omega$ resistor a long time after the switch is closed.

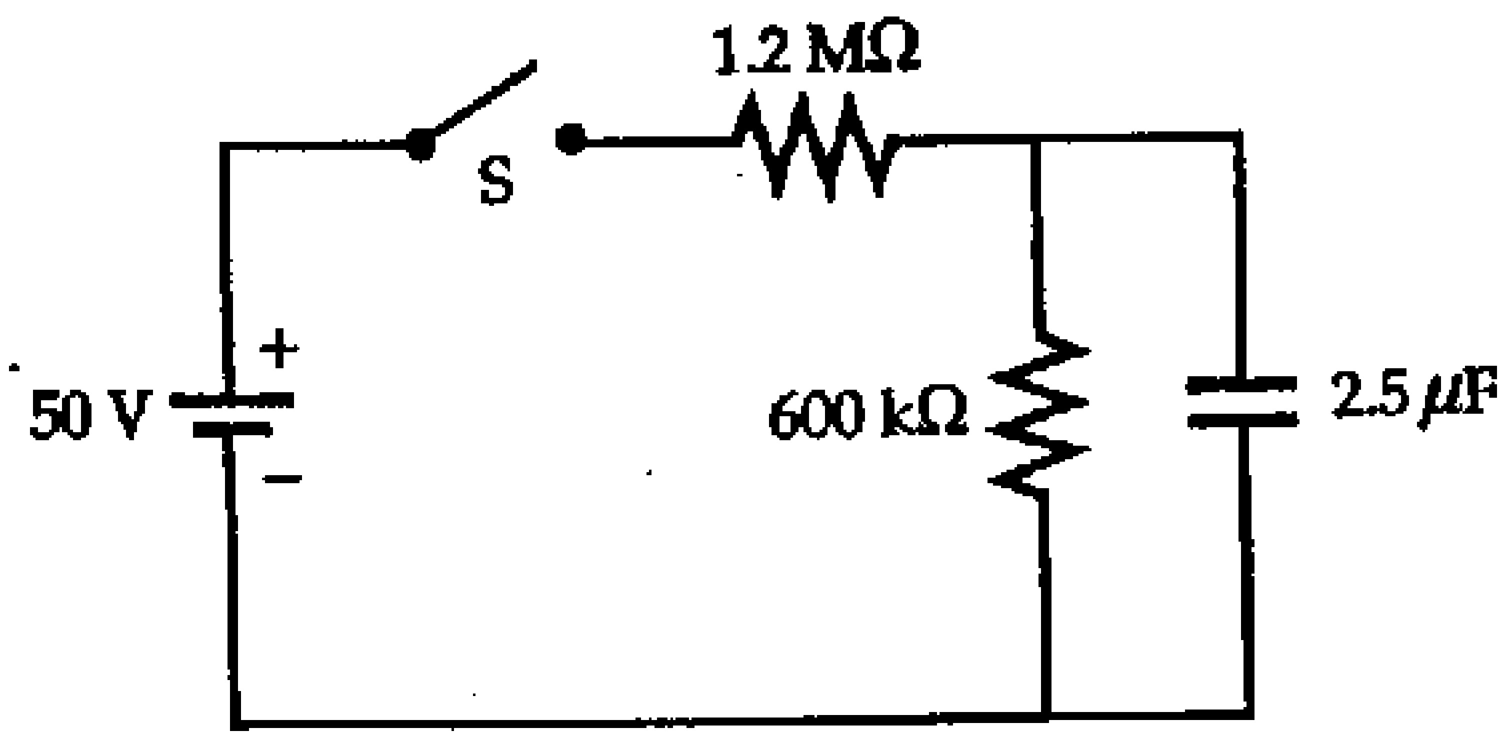
2 (d) The charge on the capacitor a long time after the switch is closed.

After the switch has been closed for a long time, it is opened. For this situation,

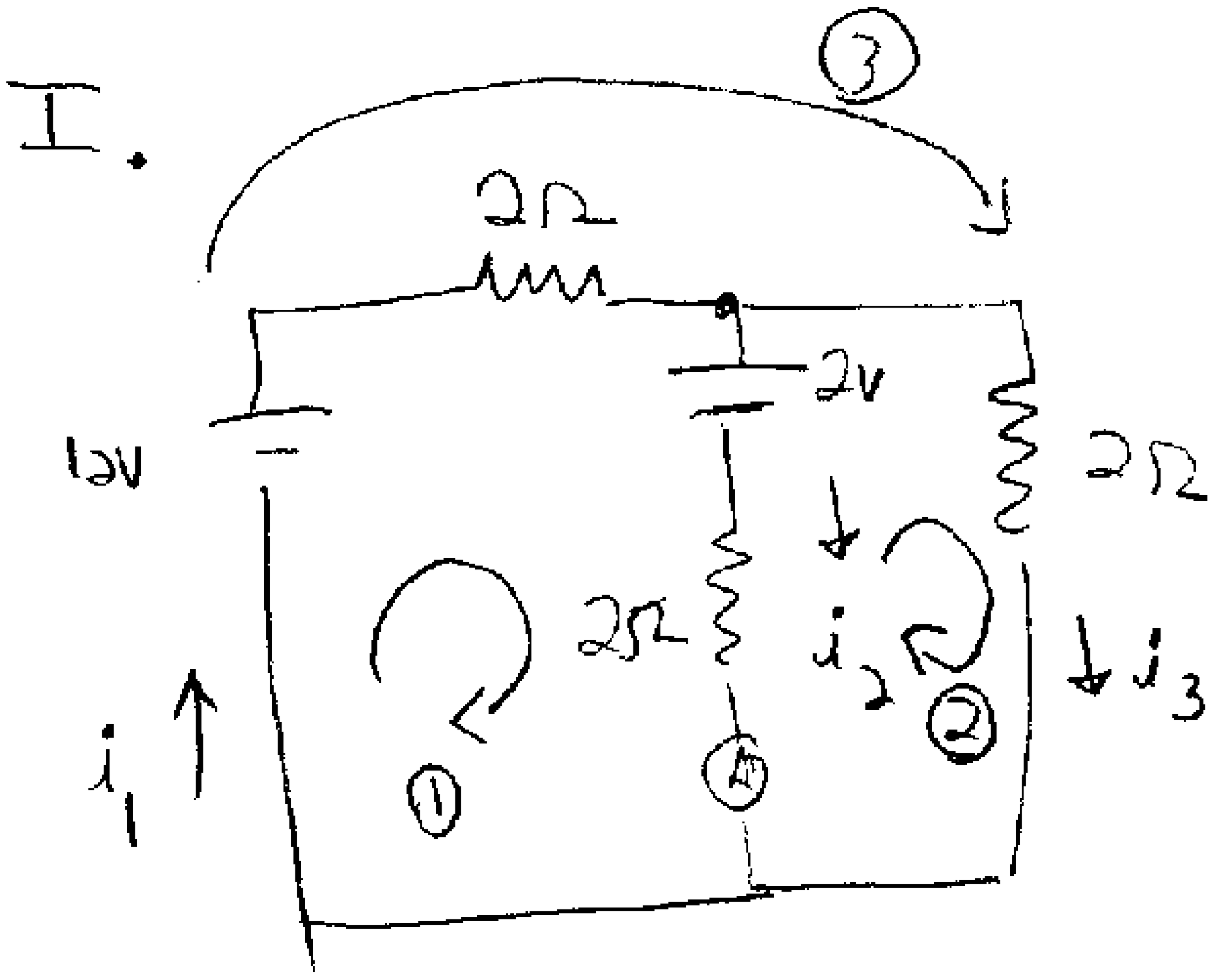
5 (e) Determine $I(t)$, the current through the 600 $k\Omega$ resistor, as a function of time.

2 (f) Sketch $I(t)$, indicating the value at $t = 0$.

3 (g) At what time after the switch is opened has the current fallen to 0.25 of its initial value?



I.



Need to set up Kirchoff's law eqns:

$i_1 = i_2 + i_3$

Loop ①: $12 - 2i_1 - 2 - 2i_2 = 0$

(Note that we traverse the 2V battery in the direction that the voltage drops.)

Loop ②: $-2i_3 + 2i_2 + 2 = 0$

Loop ③ $12 - 2i_1 - 2i_3 = 0$

(a)

Loop 1: $5 = i_1 + i_2$

Loop 3: $6 = i_1 + i_3 = i_1 + i_1 - i_2 = 2i_1 - i_2$

$$\begin{array}{rl} & 5 = i_1 + i_2 \\ + & 6 = 2i_1 - i_2 \\ \hline & 11 = 3i_1 \end{array}$$

$\boxed{i_1 = 11/3\ A}$

(up in figure)

$i_2 = 5 - i_1 = \frac{15}{3} - \frac{11}{3} = \frac{4}{3}$

$\boxed{i_2 = \frac{4}{3}\ A}$

(down in figure)

$i_3 = i_1 - i_2 = 11/3 - 4/3 = 7/3$

$\boxed{i_3 = 7/3 A}$ (down in figure)

Check with Loop 2:

$1 = i_3 - i_2 = \frac{7}{3} - \frac{4}{3} = \frac{3}{3}$ ✓

All currents are positive, so they are in the direction shown.

(b) $\mathcal{E} = IV = i_1 (12V)$

$\mathcal{E} = \left(\frac{11}{3} \text{ amps}\right)(12V)$

$$\boxed{\mathcal{E} = 44 \text{ watts}}$$

(c) $P_1 = i_1^2 R_1 = \left(\frac{11}{3}\right)^2 \cdot 2$

$P_1 = \frac{121}{9} \cdot 2 = \frac{242}{9}$

$$\boxed{P_1 = 26.89 W}$$

$P_2 = i_2^2 R_2 = \left(\frac{4}{3}\right)^2 \cdot 2$

$P_2 = \frac{32}{9}$

$$\boxed{P_2 = 3.56 W}$$

$P_3 = i_3^2 R = \left(\frac{7}{3}\right)^2 \cdot 2$

$P_3 = \frac{98}{9}$

$$\boxed{P_3 = 10.89 W}$$

Total power dissapated in the resistors = $\boxed{41.33 W = P_{TOT}}$, which is < 44 Watts

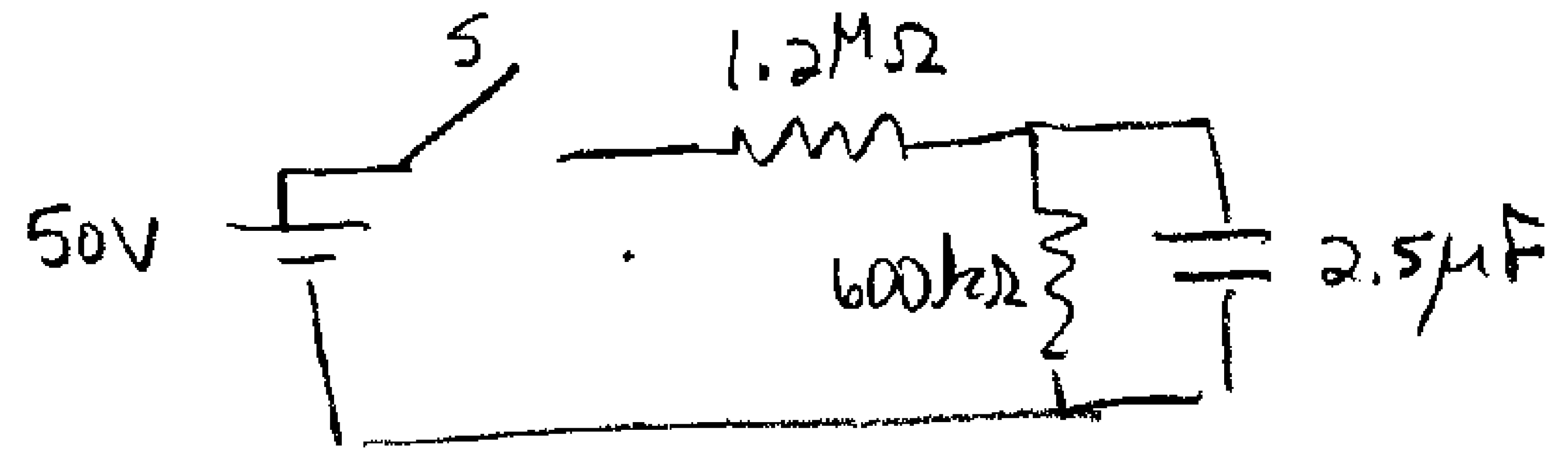
(d) The 12V battery delivers more power than is dissipated in the resistors. The additional energy is charging the 2V battery! Since i_2 passes through the 2V battery from + to −, energy is delivered to the battery.

Check: Power delived to 2V battery

$\mathcal{E}_{2V} = i_2 V = 4/3 A (2V) = \frac{8}{3} \text{ watt} = 2.67 W$,

giving a total of 44 W.

II.



(a) Immediately after S is closed, the capacitor acts like a short & the 600kΩ resistor is bypassed

$$I = \frac{50V}{1.2 \times 10^6 \Omega} = 41.7 \times 10^{-6} A$$

$$\boxed{I(t=0) = 41.7 \mu A}$$

(b) After the capacitor is charged, it acts like an open circuit

$$I = \frac{50V}{(1.2+.6)\times 10^6 \Omega} = 27.8 \mu A$$

$$\boxed{I(t=\infty) = 27.8 \mu A}$$

(c) $\Delta V_{600k\Omega} = IR = (27.8 \times 10^{-6} A)(.6 \times 10^6 \Omega) = 16.7 V$

$$\boxed{\Delta V_{600k\Omega} = 16.67 V}$$

(d) The voltage drop across the 600 kΩ resistor = voltage drop across the capacitor

$C = \frac{Q}{V}$ or $Q = CV = (2.5 \times 10^{-6} F)(16.67V)$

$$\boxed{Q = 41.67 \mu C}$$

(e) Redefine $t=0$ to be the time when the switch is opened.

When S is opened, the battery and the 1.2 MΩ resistor are removed from the circuit. The capacitor discharges through the 600 kΩ resistor.

$$\boxed{I(t) = I_0 e^{-t/\tau}}$$

$$\tau = RC = (.6\times10^{6}\,\Omega)(2.5\times10^{-6}\,F) = 1.5\,s$$

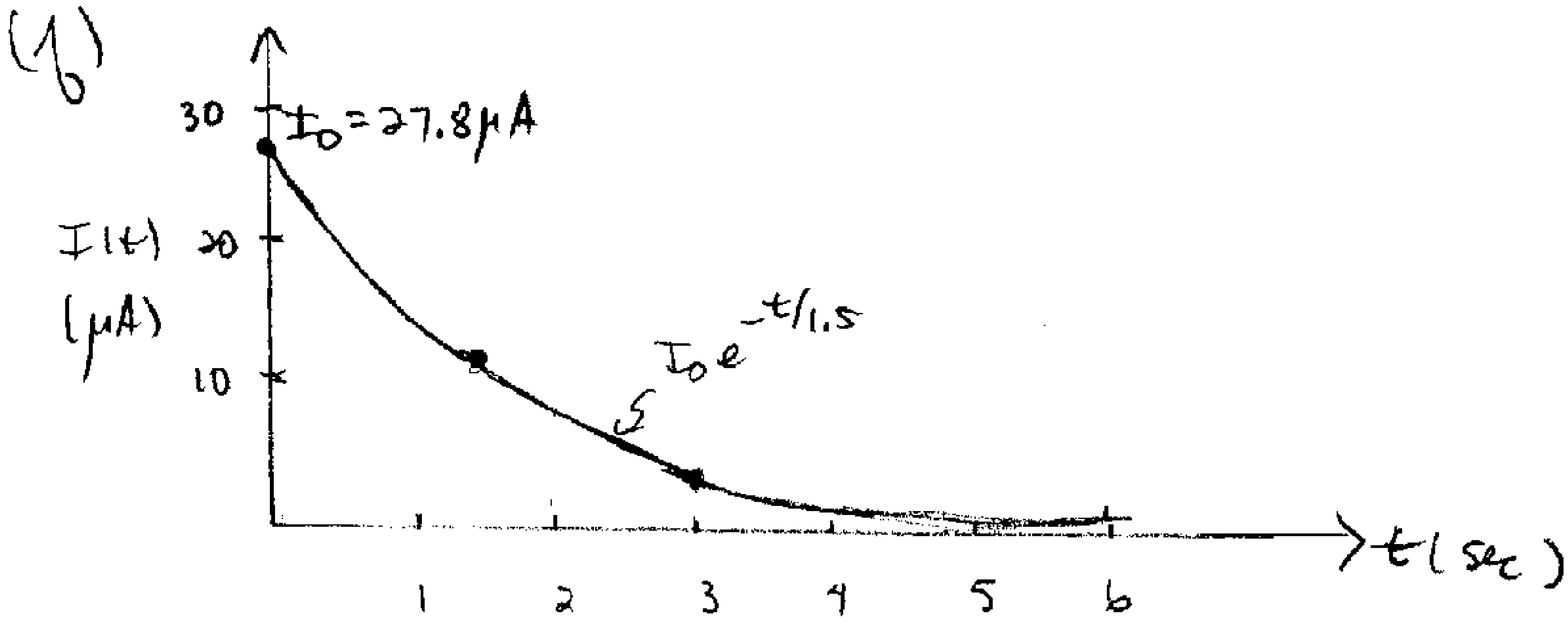
$$\boxed{\tau = 1.5\,s}$$

$$I_0 = \frac{V}{R} = \frac{16.7\,V}{.6\times10^{6}\,\Omega} = 27.8\,\mu A$$

$$\boxed{I_0 = 27.8\,\mu A}$$

$$\boxed{I(t) = 27.8\,\mu A\; e^{-t/1.5\,s}}$$

(f)



$$\boxed{t_{.25} = 2.08\,s}$$

(g) $I = I_0 e^{-t/\tau}$

$$\frac{I}{I_0} = 0.25 = e^{-t/\tau}$$

$$\ln(.25) = -t/\tau$$

$$-\ln(.25)\cdot\tau = t = 2.08\,s$$