Management of Credit Risk

- Aggregating credit exposure
  - default varies with:
  - implies what?
- Credit exposure limits to industrial sectors
- Regulatory approaches to monitoring credit risk.
Simple Models of Loan Concentration

- Migration analysis
  - Track credit rating changes within sector or pool of loans
  - Rating transition matrix reflects history of ratings changes
- Widely applied to commercial loans, credit card portfolios, and consumer loans
  - what is missing?
Rating Transition Matrix

<table>
<thead>
<tr>
<th>Risk grade: end of year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk grade: beginning</td>
<td>1</td>
<td>.85</td>
<td>.10</td>
<td>.04</td>
</tr>
<tr>
<td>of year</td>
<td>2</td>
<td>.12</td>
<td>.83</td>
<td>.03</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>.03</td>
<td>.13</td>
<td>.80</td>
</tr>
</tbody>
</table>

• Similar to matrices of ratings transitions published by S&P, Moody’s, etc.

Loan Concentration

• Concentration limits
  – On loans to individual borrower
• Concentration limit = maximum loss ÷ loss rate
  – Maximum loss expressed as percent of capital
  – Some countries, such as Chile, specify limits by sector or industry
  – FIs typically set geographic concentration limits
Portfolio Theory: Diversification

• Applying portfolio theory to loans
  – Using loans to construct the efficient frontier

• Minimum risk portfolio
  – Low risk, Low return

• Requires:
  1. Expected return on loan (typically measured by the *all-in-spread*)
  2. Loan risk
  3. Correlation of loan default risks

FI Portfolio Diversification
Portfolio Mathematics

Expected Return:

$$\bar{R}_p = \sum_{i=1}^{n} X_i \bar{R}_i$$

Variance:

$$\sigma_p^2 = \sum_{i=1}^{n} X_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_{i,j}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \rho_{i,j} \sigma_i \sigma_j$$

Moody’s KMV Portfolio Manager Model

KMV measures these as follows:

$$R_i = AIS_i - E(L_i) = AIS_i - [EDF_i \times LGD_i]$$

$$\sigma_i = UL_i = \sigma_{Di} \times LGD_i = [EDF_i(1-EDF_i)]^{1/2} \times LGD_i$$

$$\rho_{ij} = \text{correlation between systematic return components of equity returns of borrower } i \text{ and borrower } j$$
Partial Applications of Portfolio Theory

- Loan volume-based models
- Commercial bank call reports
  - Can be aggregated to estimate national allocations
- Shared national credit
  - National database that breaks commercial and industrial loan volume into 2-digit SIC codes
Partial Applications

• Loan volume-based models
  – Provide market benchmarks
• Standard deviation measure of individual FI’s loan allocations deviation from the benchmark allocations

\[
\sigma_j = \sqrt{\frac{\sum_{i=1}^{N} (X_{i,j} - X_j)^2}{N}}
\]

Loan Loss Ratio-Based Models

• Estimate loan loss risk by SIC sector
  – Time-series regression:

\[
\frac{[\text{sectoral losses in } i\text{th sector}]}{[\text{loans to } i\text{th sector}]} = \alpha + \beta_i \frac{[\text{total loan losses}]}{[\text{total loans}]}\]

• Usual problems with econometric models
  – specification, data, backward looking, etc.
Regulatory Models

- Credit concentration risk evaluation largely subjective and based on examiner discretion
  - ridiculous: sorry!
- Quantitative models rejected by regulators
  - the methods were not sufficiently advanced and available data were not sufficient
- Subjective assessment
  - how to miss the subprime/real-estate crisis

CreditMetrics

- If next year is a bad year, how much will I lose on my loans and loan portfolio?
  \[ \text{VAR} = P \times 1.65 \times \sigma \]
- Neither \( P \), nor \( \sigma \) observed: estimate
- Calculated using:
  1. Data on borrower’s credit rating
  2. Rating transition matrix
  3. Recovery rates on defaulted loans
  4. Yield spreads
Credit Risk+

• Developed by Credit Suisse Financial Products
• Based on insurance literature:
  – Losses reflect frequency of event and severity of loss
• Loan default is random
• Loan default probabilities are independent
• Appropriate for large portfolios of small loans
• Modeled by a Poisson distribution

Credit Risk+ Model: Determinants of Loan Losses

![Diagram of Credit Risk+ Model: Determinants of Loan Losses]

- Frequency of Defaults
- Severity of Losses
- Distribution of Default Losses
Modelling Data Inputs - Summary

Credit Grading:

PD  LGD  EAD  Correlations

CR Measurement

1. Empirical
2. Country and bank specific

Modelled by facility

1. Traditional (RAgencies, Z-scores, ANNs)
2. Modern:
   • Structural
   • Reduced Form

Empirical:
• Fixed
• Equity based
• Mapped from industries

Why the Portfolio Focus Matters

• Traditionally, portfolio managers have relied on their intuitive “feel” for concentration;
• This ignores basic rationale for being in the finance business – relationship between risk and return;
• Portfolio approach allows portfolio manager to re-cast credit lines in terms of contribution to “Marginal Portfolio Volatility”
Default Mode Modelling

• MTM models focus on the probabilities of being in either of two states at the relevant time horizon - default or non-default

• Key to the default mode model is the separate use of PD and LGD in the calculation of Expected Loss $EL$ and Unexpected Loss $UL$

• This is the level of complexity envisaged by the Basel II reforms

Losses in Default Mode

At the heart of the default mode models is the calculation of expected loss and the volatility of expected loss:

\[
EL = EAD \times PD \times LGD \\
UL = \sqrt{EL(EL \times LGD - EL)}
\]

Where:
- EL is expected loss;
- UL is unexpected loss;

WHY??
Portfolio Credit Risk

- Practice is to group risks by facility type
- Then calculate correlation ($\rho_i$ for facility i) between the default rates of each facility group and that of the portfolio as a whole
- Then calculate for the portfolio:

\[
EL_p = \sum_i EL_i \\
UL_p = \sum_i UL_i \rho_i
\]

Example

- A bank has the following 3-facility portfolio, - PDs, EADs and LGDs are as shown
- Calculate the expected loss and risk characteristics of the portfolio

<table>
<thead>
<tr>
<th>Type of Facility</th>
<th>Nominal principal</th>
<th>Risk Grade</th>
<th>EAD</th>
<th>PD</th>
<th>LGD</th>
<th>Correlation with Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-yr Loan</td>
<td>$2,777,778</td>
<td>A3</td>
<td>$3,000,000</td>
<td>.09%</td>
<td>60%</td>
<td>.1</td>
</tr>
<tr>
<td>10-yr IRS</td>
<td>$5,000,000</td>
<td>AA</td>
<td>$397,283</td>
<td>.03%</td>
<td>40%</td>
<td>.2</td>
</tr>
<tr>
<td>15-yr Mortgages</td>
<td>$4,000,000</td>
<td>B</td>
<td>$300,772</td>
<td>13.00%</td>
<td>70%</td>
<td>.15</td>
</tr>
</tbody>
</table>
Calculating Individual Risks

- Given the figures in the example, we can calculate:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>EL(1)</td>
<td>3m x .09% x .6</td>
<td>$</td>
<td>1,620</td>
</tr>
<tr>
<td>EL(2)</td>
<td>397,283 x .03% x .4</td>
<td>$</td>
<td>48</td>
</tr>
<tr>
<td>EL(3)</td>
<td>300,772 x 13% x .7</td>
<td>$</td>
<td>27,370</td>
</tr>
<tr>
<td>EL(p)</td>
<td>sum(ELi)</td>
<td>$</td>
<td>29,038</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>UL(1)</td>
<td>$</td>
<td>53,976</td>
</tr>
<tr>
<td>UL(2)</td>
<td>$</td>
<td>2,752</td>
</tr>
<tr>
<td>UL(3)</td>
<td>$</td>
<td>70,805</td>
</tr>
</tbody>
</table>

Calculating Portfolio Risk

- Portfolio unexpected loss is the weighted sum of the individual unexpected losses:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Contribution (1)</td>
<td>UL(1) x Corr 1</td>
<td>$</td>
<td>5,398</td>
</tr>
<tr>
<td>Contribution (2)</td>
<td>UL(2) x Corr 2</td>
<td>$</td>
<td>550</td>
</tr>
<tr>
<td>Contribution (3)</td>
<td>UL(3) x Corr 3</td>
<td>$</td>
<td>10,621</td>
</tr>
<tr>
<td>UL(p)</td>
<td></td>
<td>$</td>
<td>16,569</td>
</tr>
</tbody>
</table>

- Portfolio risk is a multiple of this depending on the shape of the compound distribution and risk tolerance
A Note on Credit Diversification

• Unlike market risk, default correlations tend to be very low in credit risk
• E.g. in a typical stock market portfolio, 15 - 20 shares is sufficient to gain most of the benefits of diversification
• In comparison, in a credit portfolio the empirical evidence suggests that there almost always gains from further diversification

Mark-to-Market Modelling

• MTM models define credit events to encompass not only default, but migration to any credit rating other than the current one
• By valuing every credit in every possible state and then probability weighting them, the MTM model effectively simulates the price at which any credit could be sold - hence the MTM label
e.g. Credit Migrations from BBB

Range of possible credit ratings at the end of the year - each has an associated probability of occurring:

<table>
<thead>
<tr>
<th>Year-end Rating</th>
<th>Probability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.02</td>
</tr>
<tr>
<td>AA</td>
<td>0.33</td>
</tr>
<tr>
<td>A</td>
<td>5.95</td>
</tr>
<tr>
<td>BBB</td>
<td>86.93</td>
</tr>
<tr>
<td>BB</td>
<td>5.30</td>
</tr>
<tr>
<td>B</td>
<td>1.17</td>
</tr>
<tr>
<td>CCC</td>
<td>0.12</td>
</tr>
<tr>
<td>Default</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Note: In the default mode all we needed was the PD = .18

Measuring Risk in MTM Models

- MTM models value each individual credit exposure in each possible migratory state
- Risk is then measured by considering the entire distribution of possible outcomes of value across all credits, taking into account their joint probabilities
- This involves a massive computational exercise to construct a distribution covering all possible outcomes
- For example, with 8 credit grades (including default) even 2 credits involve 64 possible outcomes – each with a separate probability
MTM Models - Strengths and Weaknesses

• Strengths:
  • Account for all changes of credit rating (not just default)
  • Better replicate reality

• Weakness - ahead of their time:
  • The models demand data that are not yet widely available
  • They require knowledge about obligors that is often not readily available
  • Where information or data are not available they require heroic assumptions
  • They simulate market values where markets typically don’t exist

• They are nevertheless the way of the future

Pertinent Websites

Bank for International Settlements  www.bis.org
Federal Reserve Bank  www.federalreserve.gov
Moody’s  www.moodys.com
Moody’s KMV  www.moodyskmv.com
National Association of Insurance Commissioners  www.naic.org
Standard & Poors  www.standardandpoors.com