Chapter 11: Credit Risk – Loan Pricing and Terms

9. County Bank offers one-year loans with a stated rate of 9 percent but requires a compensating balance of 10 percent. What is the true cost of this loan to the borrower? How does the cost change if the compensating balance is 15 percent? If the compensating balance is 20 percent? In each case, assume origination fees and the reserve requirement are zero.

The true cost is the loan rate ÷ (1 – compensating balance rate) = 9% ÷ (1.0 – 0.1) = 10 percent. For compensating balance rates of 15 percent and 20 percent, the true cost of the loan would be 10.59 percent and 11.25 percent, respectively. Note that as the compensating balance rate increases by a constant amount, the true cost of the loan increases at an increasing rate.

10. Metrobank offers one-year loans with a 9 percent stated or base rate, charges a 0.25 percent loan origination fee, imposes a 10 percent compensating balance requirement, and must pay a 6 percent reserve requirement to the Federal Reserve. The loans typically are repaid at maturity.

a. If the risk premium for a given customer is 2.5 percent, what is the simple promised interest return on the loan?

The simple promised interest return on the loan is \( BR + m = 0.09 + 0.025 = 0.115 \) or 11.5%.

b. What is the contractually promised gross return on the loan per dollar lent?

\[
1 + k = 1 + \frac{0.0025 + (0.09 + 0.025)}{1 - 0.1(1 - 0.06)} = 1 + \frac{0.1175}{0.906} = 1.1297 \text{ or } k = .1297 = 12.97\%
\]

c. Which of the fee items has the greatest impact on the gross return?

The compensating balance has the strongest effect on the gross return on the loan. Without the compensating balance, the gross return would equal 11.75 percent, a reduction of 1.22 percent. Without the origination fee, the gross return would be 12.69 percent, a reduction of only 0.28 percent. Eliminating the reserve requirement would cause the gross return to increase to 13.06 percent, an increase of 0.09 percent.
11. Why are most retail borrowers charged the same rate of interest, implying the same risk premium or class? What is credit rationing? How is it used to control credit risks with respect to retail and wholesale loans?

Most retail loans are small in size relative to the overall investment portfolio of an FI, and the cost of collecting information on household borrowers is high. As a result, most retail borrowers are charged the same rate of interest that implies the same level of risk.

Credit rationing involves restricting the amount of loans that are available to individual borrowers. On the retail side, the amount of loans provided to borrowers may be determined solely by the proportion of loans desired in this category rather than price or interest rate differences, thus the actual credit quality of the individual borrowers. On the wholesale side, the FI may use both credit quantity and interest rates to control credit risk. Typically, more risky borrowers are charged a higher risk premium to control credit risk. However, the expected returns from increasingly higher interest rates that reflect higher credit risk at some point will be offset by higher default rates. Thus, rationing credit through quantity limits will occur at some interest rate level even though positive loan demand exists at even higher risk premiums.

15. Why is the degree of collateral as specified in the loan agreement of importance to the lender? If the book value of the collateral is greater than or equal to the amount of the loan, is the credit risk of the lender fully covered? Why, or why not?

Collateral provides the lender with some assets that can be used against the amount of the loan in the case of default. However, collateral has value only to the extent of its market value, and thus a loan fully collateralized at book value may not be fully collateralized at market value. Further, errors in the recording of collateralized positions may limit or severely reduce the protected positions of a lender.

Chapter 11: Credit Risk – Credit Scoring and Default

18. Suppose the estimated linear probability model used by an FI to predict business loan applicant default probabilities is \( PD = 0.03X_1 + 0.02X_2 - 0.05X_3 + \text{error} \), where \( X_1 \) is the borrower's debt/equity ratio, \( X_2 \) is the volatility of borrower earnings, and \( X_3 = 0.10 \) is the borrower’s profit ratio. For a particular loan applicant, \( X_1 = 0.75 \), \( X_2 = 0.25 \), and \( X_3 = 0.10 \).

a. What is the projected probability of default for the borrower?

\[
PD = 0.03(0.75) + 0.02(0.25) - 0.05(0.10) = 0.0225
\]

b. What is the projected probability of repayment if the debt/equity ratio is 2.5?

\[
PD = 0.03(2.5) + 0.02(0.25) - 0.05(0.10) = 0.075
\]

The expected probability of repayment is \( 1 - 0.075 = 0.925 \).

The major weakness of the linear probability model?
A major weakness of this model is that the estimated probabilities can be below 0.0 or above 1.0, an occurrence that does not make economic or statistical sense.

20. MNO, Inc., a publicly traded manufacturing firm in the United States, has provided the following financial information in its application for a loan. All numbers are in thousands of dollars.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities and Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>Accounts payable</td>
</tr>
<tr>
<td>$20</td>
<td>$30</td>
</tr>
<tr>
<td>Accounts receivables</td>
<td>Notes payable</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Inventory</td>
<td>Accruals</td>
</tr>
<tr>
<td>90</td>
<td>30</td>
</tr>
<tr>
<td>Plant and equipment</td>
<td>Long-term debt</td>
</tr>
<tr>
<td>500</td>
<td>150</td>
</tr>
<tr>
<td>Total assets</td>
<td>Equity (ret. earnings = $0)</td>
</tr>
<tr>
<td>$700</td>
<td>400</td>
</tr>
</tbody>
</table>

Also assume sales = $500,000, cost of goods sold = $360,000, taxes = $56,000, interest payments = $40,000, net income = $44,000, the dividend payout ratio is 50 percent, and the market value of equity is equal to the book value.

a. What is the Altman discriminant function value for MNO, Inc.? Recall that:

Net working capital = Current assets - Current liabilities.

Current assets = Cash + Accounts receivable + Inventories.

Current liabilities = Accounts payable + Accruals + Notes payable.

EBIT = Revenues - Cost of goods sold - Depreciation.

Net income = (EBIT – Interest) (Tax rate).

Retained earnings = Net income (1 - Dividend payout ratio)

Altman’s discriminant function is given by: 
\[ Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5 \]

All numbers are in $000s.

\[ X_1 = (20+90+30-30-90-30) / 700 = .0714 \quad X_1 = \text{Working capital/total assets (TA)} \]

\[ X_2 = 44(1-.5) / 700 = .0314 \quad X_2 = \text{Retained earnings/TA} \]

\[ X_3 = (500-360) / 700 = .20 \quad X_3 = \text{EBIT/TA} \]

\[ X_4 = 400 / 150 = 2.6667 \quad X_4 = \text{Market value of equity/Book value of long-term debt} \]
FIN 683                 Financial-Institutions Management
Professor Robert Hauswald                                                      Kogod School of Business, AU

\[ X_5 = \frac{500}{700} = .7143 \]

\[ Z = 1.2(0.0714) + 1.4(0.0314) + 3.3(0.20) + 0.6(2.6667) + 1.0(0.7143) = 3.104 \]

\[ X_5 = \text{Sales/TA} \]

b. Should you approve MNO, Inc.’s application to your bank for a $500,000 capital expansion loan?

Since the Z-score of 3.104 is greater than 2.99, ABC Inc.’s application for a capital expansion loan should be approved.

c. If sales for MNO were $300,000, the market value of equity was only half of book value, and the cost of goods sold, interest, and tax rate were unchanged, what would be the net income for MNO? Assume the tax credit can be used to offset other tax liabilities incurred by other divisions of the firm. Would your credit decision change?

ABC’s net income would be -$100,000 without taking into account tax credits. Note, that ABC’s tax liability is -$56,000. If we assume that ABC uses this tax credit against other tax liabilities, then (numbers are in $000s):

\[ X_1 = \frac{(20 + 90 + 90 - 30 - 90 - 30)}{700} = .0714 \]

\[ X_2 = \frac{-44}{700} = -0.0629 \]

\[ X_3 = \frac{-60}{700} = -0.0857 \]

\[ X_4 = \frac{200}{150} = 1.3333 \]

\[ X_5 = \frac{300}{700} = 0.4286 \]

\[ Z = 1.2(0.0714) + 1.4(-0.0629) + 3.3(-0.0857) + 0.6(1.3333) + 1.0(0.4286) = 0.9434 \]

Since ABC’s Z-score falls to 0.9434 < 1.81, credit should be denied.

d. Would the discriminant function change for firms in different industries? Would the function be different for manufacturing firms in different geographic sections of the country? What are the implications for the use of these types of models by FIs?

Discriminant function models are very sensitive to the weights for the different variables. Since different industries have different operating characteristics, a reasonable answer would be yes with the condition that there is no reason that the functions could not be similar for different industries. In the retail market, the demographics of the market play a big role in the value of the weights. For example, credit card companies often evaluate different models for different areas of the country. Because of the sensitivity of the models, extreme care should be taken in the process of selecting the correct sample to validate the model for use.
22. If the rate on one-year T-Bills currently is 6 percent, what is the repayment probability for each of the following two securities? Assume that if the loan is defaulted, no payments are expected. What is the market-determined risk premium for the corresponding probability of default for each security?

a. One-year AA-rated bond yielding 9.5 percent.

Probability of repayment \( p = \frac{1 + i}{1 + k} \)

For an AA-rated bond \( = \frac{1 + .06}{1 + .095} = 0.968 \), or 96.80 percent

The market determined risk premium is \( 0.095 - 0.060 = 0.035 \) or 3.5 percent

b. One-year BB-rated bond yielding 13.5 percent.

Probability of repayment \( p = \frac{1 + i}{1 + k} \)

For BB-rated bond \( = \frac{1 + .06}{1 + .135} = 93.39 \) percent

The market determined risk premium is \( 0.135 - 0.060 = 0.075 \) or 7.5 percent

23. A bank has made a loan charging a base lending rate of 10 percent. It expects a probability of default of 5 percent. If the loan is defaulted, the bank expects to recover 50 percent of its money through the sale of its collateral. What is the expected return on this loan?

\[
E(r) = p(1 + k) + (1 - p)(1 + k)(\gamma) \]  
where \( \gamma \) is the percentage generated when the loan is defaulted. 

\[
E(r) = .95(1 + .10) + .05(1 + .10)(.50) = 1.0450 + .0275 = 1.0725 - 1.0 = 7.25\%
\]

30. The table below shows the dollar amounts of outstanding bonds and corresponding default amounts for every year over the past five years. Note that the default figures are in millions, while those outstanding are in billions. The outstanding figures reflect default amounts and bond redemptions.

<table>
<thead>
<tr>
<th>Years after Issuance</th>
<th>Loan Type</th>
<th>1 Year</th>
<th>2 Years</th>
<th>3 Years</th>
<th>4 Years</th>
<th>5 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A-rated: Annual default (millions)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$1</td>
<td>$2</td>
</tr>
<tr>
<td></td>
<td>Outstanding (billions)</td>
<td>$100</td>
<td>$95</td>
<td>$93</td>
<td>$91</td>
<td>$88</td>
</tr>
<tr>
<td></td>
<td>B-rated: Annual default (millions)</td>
<td>0</td>
<td>$1</td>
<td>$2</td>
<td>$3</td>
<td>$4</td>
</tr>
<tr>
<td></td>
<td>Outstanding (billions)</td>
<td>$100</td>
<td>$94</td>
<td>$92</td>
<td>$89</td>
<td>$85</td>
</tr>
</tbody>
</table>
C-rated: Annual default (millions) $1 $3 $5 $5 $6
Outstanding (billions) $100 $97 $90 $85 $79

a. What are the annual and cumulative default rates of the above bonds?

<table>
<thead>
<tr>
<th>Year</th>
<th>Default</th>
<th>Balance</th>
<th>Annual</th>
<th>Survival =</th>
<th>Cumulative</th>
<th>% Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Default</td>
<td>1 - An. Def.</td>
<td>Default Rate</td>
<td>Default Rate</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>100,000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.0000%</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>95,000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.0000%</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>93,000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.0000%</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>91,000</td>
<td>0.000011</td>
<td>0.999989</td>
<td>0.000011</td>
<td>0.0011%</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>88,000</td>
<td>0.000023</td>
<td>0.999977</td>
<td>0.000034</td>
<td>0.0034%</td>
</tr>
</tbody>
</table>

Where cumulative default for nth year = 1 - product of survival rates to that year.

B-rated Bonds

<table>
<thead>
<tr>
<th>Year</th>
<th>Default</th>
<th>Balance</th>
<th>Annual</th>
<th>Survival =</th>
<th>Cumulative</th>
<th>% Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Default</td>
<td>1 - An. Def.</td>
<td>Default Rate</td>
<td>Default Rate</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>100,000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.0000%</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>94,000</td>
<td>0.000011</td>
<td>0.999989</td>
<td>0.000011</td>
<td>0.0011%</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>92,000</td>
<td>0.000022</td>
<td>0.999978</td>
<td>0.000032</td>
<td>0.0032%</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>89,000</td>
<td>0.000034</td>
<td>0.999966</td>
<td>0.000066</td>
<td>0.0066%</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>85,000</td>
<td>0.000047</td>
<td>0.999953</td>
<td>0.000113</td>
<td>0.0113%</td>
</tr>
</tbody>
</table>

C-rated Bonds

<p>| Millions | Millions | Annual | Survival = | Cumulative | % Cumulative |
|----------|----------|--------|-------------|------------|--------------|--------------|</p>
<table>
<thead>
<tr>
<th>Year</th>
<th>Default</th>
<th>Balance</th>
<th>Default</th>
<th>1 - An. Def.</th>
<th>Default Rate</th>
<th>Default Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>100,000</td>
<td>0.000100</td>
<td>0.999900</td>
<td>0.000100</td>
<td>0.0100%</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>97,000</td>
<td>0.000310</td>
<td>0.999690</td>
<td>0.000041</td>
<td>0.0041%</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>90,000</td>
<td>0.000560</td>
<td>0.999444</td>
<td>0.000096</td>
<td>0.0096%</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>85,000</td>
<td>0.000590</td>
<td>0.999410</td>
<td>0.000155</td>
<td>0.0155%</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>79,000</td>
<td>0.000760</td>
<td>0.999240</td>
<td>0.000231</td>
<td>0.0231%</td>
</tr>
</tbody>
</table>

### Years after Issuance

<table>
<thead>
<tr>
<th>Bond Type</th>
<th>1 Year</th>
<th>2 Years</th>
<th>3 Years</th>
<th>4 Years</th>
<th>5 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-rated: Yearly default</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0.0011%</td>
<td>0.0023%</td>
</tr>
<tr>
<td>Cumulative default</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0.0011%</td>
<td>0.0034%</td>
</tr>
<tr>
<td>B-rated: Yearly default</td>
<td>0%</td>
<td>0.0011%</td>
<td>0.0022%</td>
<td>0.0034%</td>
<td>0.0047%</td>
</tr>
<tr>
<td>Cumulative default</td>
<td>0%</td>
<td>0.0011%</td>
<td>0.0032%</td>
<td>0.0066%</td>
<td>0.0113%</td>
</tr>
<tr>
<td>C-rated: Yearly default</td>
<td>0.0010%</td>
<td>0.0031%</td>
<td>0.0056%</td>
<td>0.0059%</td>
<td>0.0076%</td>
</tr>
<tr>
<td>Cumulative default</td>
<td>0.0010%</td>
<td>0.0041%</td>
<td>0.0096%</td>
<td>0.0155%</td>
<td>0.0231%</td>
</tr>
</tbody>
</table>

Note: These percentage values seem very small. More reasonable values can be obtained by increasing the default dollar values by a factor of ten, or by decreasing the outstanding balance values by a factor of 0.10. Either case will give the same answers that are shown below. While the percentage numbers seem somewhat more reasonable, the true values of the problem are (a) that default rates are higher on lower rated assets, and (b) that the cumulative default rate involves more than the sum of the annual default rates.

### C-rated Bonds

<table>
<thead>
<tr>
<th>Year</th>
<th>Default</th>
<th>Balance</th>
<th>Default</th>
<th>1 - An. Def.</th>
<th>Default Rate</th>
<th>Default Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>100,000</td>
<td>0.000100</td>
<td>0.999900</td>
<td>0.000100</td>
<td>0.0100%</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>97,000</td>
<td>0.000309</td>
<td>0.999691</td>
<td>0.000409</td>
<td>0.0409%</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>90,000</td>
<td>0.000556</td>
<td>0.999444</td>
<td>0.000965</td>
<td>0.0965%</td>
</tr>
</tbody>
</table>
Chapter 11: Credit Risk – RAROC

32. A bank is planning to make a loan of $5,000,000 to a firm in the steel industry. It expects to charge a servicing fee of 50 basis points. The loan has a maturity of 8 years with a duration of 7.5 years. The cost of funds (the RAROC benchmark) for the bank is 10 percent. The bank has estimated the maximum change in the risk premium on the steel manufacturing sector to be approximately 4.2 percent, based on two years of historical data. The current market interest rate for loans in this sector is 12 percent.

a. Using the RAROC model, determine whether the bank should make the loan?

RAROC = Fees and interest earned on loan/Loan or capital risk

Loan risk, or \( \Delta LN \) = \(-D_{LN} \times LN \times (\Delta R/(1 + R))\) = \(-7.5 \times \$5m \times (0.042/1.12)\) = \(-$1,406,250\)

Expected interest = 0.12 \times $5,000,000 = $600,000

Servicing fees = 0.0050 \times $5,000,000 = $25,000

Less cost of funds = 0.10 \times $5,000,000 = -$500,000

Net interest and fee income = $125,000

RAROC = $125,000/$1,406,250 = 8.89 percent. Since RAROC is lower than the cost of funds to the bank, the bank should not make the loan.

b. What should be the duration in order for this loan to be approved?

For RAROC to be 10 percent, loan risk should be:

\[
$125,000/\Delta LN = 0.10 \quad \Rightarrow \Delta LN = 125,000 / 0.10 = 1,250,000
\]

\[
\Rightarrow -D_{LN} \times LN \times (\Delta R/(1 + R)) = 1,250,000
\]

\[
D_{LN} = 1,250,000/(5,000,000 \times (0.042/1.12)) = 6.67 \text{ years.}
\]

Thus, this loan can be made if the duration is reduced to 6.67 years from 7.5 years.
c. Assuming that duration cannot be changed, how much additional interest and fee income will be necessary to make the loan acceptable?

Necessary RAROC = Income/Risk ⇒ Income = RAROC x Risk

\[ \text{Income} = 1,406,250 \times 0.10 = 140,625 \]

Therefore, additional income = $140,625 - $125,000 = $15,625.

d. Given the proposed income stream and the negotiated duration, what adjustment in the loan rate would be necessary to make the loan acceptable?

Need an additional $15,625 ⇒ $15,625/$5,000,000 = 0.003125 or .3125%

Expected interest = 0.123125 x $5,000,000 = $615,625
Servicing fees = 0.0050 x $5,000,000 = $25,000
Less cost of funds = 0.10 x $5,000,000 = -$500,000
Net interest and fee income = $140,625

RAROC = $140,625/1,406,250 = 10.00 percent = cost of funds to the bank. Thus, increasing the loan rate to 12.3125% will make the loan acceptable.

Chapter 11: Credit Risk – Option Models

33. A firm is issuing a two-year debt in the amount of $200,000. The current market value of the assets is $300,000. The risk-free rate is 4 percent and the standard deviation of the rate of change in the underlying assets of the borrower is 20 percent. Using an options framework, determine the following:

a. The current market value of the loan.

b. The risk premium to be charged on the loan.

The following need to be estimated first: \( d, h_1 \) and \( h_2 \).

\[ d = B e^{-i \tau / A} = \frac{200,000 e^{-0.04(2)}}{300,000} = 0.6154 \text{ or } 61.54 \text{ percent.} \]

\[ h_1 = -\left[ 0.5 \times (0.2)^2 \times 2 - \ln(0.6154) \right] / (0.2)(2)^{1/2} = -1.8578 \]

\[ h_2 = -\left[ 0.5 \times (0.2)^2 \times 2 + \ln(0.6154) \right] / (0.2)(2)^{1/2} = 1.5750 \]
Current market value of loan  

\[ l(\tau) = B e^{i \tau} [N(h_1)1/d + N(h_2)] \]

\[ = $184,623.27[N(-1.8578) x 1.62493 + N(1.5750)] \]

\[ = $184,623.27[1.62493 x 0.031654 + 0.94265] = $183,531 \]

The risk premium \[ k(\tau) - i = (-1/\tau) \ln[N(h_2) + (1/d)N(h_1)] \]

\[ = (-\frac{1}{\tau}) \ln[0.94265 + 1.62493 \times 0.031654] = 0.002966 = 0.2966\% \]

34. A firm has assets of $200,000 and total debts of $175,000. With an option pricing model, the implied volatility of the firm’s assets is estimated at $10,730. Under the KMV method, what is the expected default frequency (assuming a normal distribution for assets)?

The firm will be in technical bankruptcy if the value of the assets fall below $175,000. If \( \sigma = 10,730 \), then it takes \( 25,000/10,730 = 2.33 \) standard deviations for the assets to fall below this value. Under the assumption that the market value of the assets are normally distributed, then 2.33 represents a 1 percent probability that the firm will become bankrupt.

35. Carman County Bank (CCB) has a $5 million face value outstanding, adjustable-rate loan to a company that has a leverage ratio of 80 percent. The current risk free rate is 6 percent, and the time to maturity on the loan is exactly ½ year. The asset risk of the borrower, as measured by the standard deviation of the rate of change in the value of the underlying assets, is 12 percent. The normal density function values are given below:

<table>
<thead>
<tr>
<th>h</th>
<th>N(h)</th>
<th>h</th>
<th>N(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.55</td>
<td>0.0054</td>
<td>2.50</td>
<td>0.9938</td>
</tr>
<tr>
<td>-2.60</td>
<td>0.0047</td>
<td>2.55</td>
<td>0.9946</td>
</tr>
<tr>
<td>-2.65</td>
<td>0.0040</td>
<td>2.60</td>
<td>0.9953</td>
</tr>
<tr>
<td>-2.70</td>
<td>0.0035</td>
<td>2.65</td>
<td>0.9960</td>
</tr>
<tr>
<td>-2.75</td>
<td>0.0030</td>
<td>2.70</td>
<td>0.9965</td>
</tr>
</tbody>
</table>

a. Use the Merton option valuation model to determine the market value of the loan.

The following need to be estimated first: \( d, h_1 \) and \( h_2 \).

\[ D = .80 \]

\[ h_1 = -[0.5 \times (0.12)^2 \times 0.5 - \ln(0.8)]/0.12\times\sqrt{0.5} = -0.226744/0.084853 = -2.6722 \]
\[ h_2 = -(0.5 \times (0.12)^2 \times 0.5 + \ln(0.8)) / (0.12) \sqrt{0.5} = 0.219544 / 0.084853 = 2.5873 \]

Current market value of loan
\[ = l(\tau) = Be^{i\tau} [N(h_1)1/d + N(h_2)] \]
\[ = $4,852,227.67[N(-2.6722) \times 1.25 + N(2.5873)] \]
\[ = $4,852,227.67 [1.25 \times 0.003778 + 0.995123] \]
\[ = $4,851,478 \]

b. What should be the interest rate for the last six months of the loan?

The risk premium \( k(\tau) - i = (-1/\tau) \ln[N(h_2) + (1/d)N(h_1)] \)
\[ = (-1/0.5)\ln[0.995123 + 1.25 \times 0.003778] = 0.0003 \]
The loan rate = risk-free rate plus risk premium = 0.06 + 0.0003 = 0.0603 or 6.03%.

Chapter 12: Loan Portfolio Risk – Mean-Variance Portfolio Risk

7. The Bank of Tinytown has two $20,000 loans that have the following characteristics: Loan A has an expected return of 10 percent and a standard deviation of returns of 10 percent. The expected return and standard deviation of returns for loan B are 12 percent and 20 percent, respectively.

a. If the correlation coefficient between loans A and B is .15, what are the expected return and standard deviation of this portfolio?

\[ X_A = X_B = 20,000 / 40,000 = .5 \]

Expected return = 0.5(10%) + 0.5(12%) = 11 percent

Standard deviation = \[ [0.5^2(0.10)^2 + 0.5^2(0.20)^2 + 2(0.5)(0.5)(0.10)(0.20)(.15)]^{1/2} = 11.83 \text{ percent} \]

b. What is the standard deviation of the portfolio if the correlation is -.15?

Standard deviation = \[ [0.5^2(0.10)^2 + 0.5^2(0.20)^2 + 2(0.5)(0.5)(0.10)(0.20)(-0.15)]^{1/2} = 10.49 \text{ percent} \]

c. What role does the covariance, or correlation, play in the risk reduction attributes of modern portfolio theory?
The risk of the portfolio as measured by the standard deviation is reduced when the covariance is reduced. If the correlation is less than +1.0, the standard deviation of the portfolio will always be less than the weighted average of the standard deviations of the individual assets.

11. A bank vice president is attempting to rank, in terms of the risk-reward trade-off, the loan portfolios of three loan officers. Information on the portfolios is noted below. How would you rank the three portfolios?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10%</td>
<td>8%</td>
</tr>
<tr>
<td>B</td>
<td>12%</td>
<td>9%</td>
</tr>
<tr>
<td>C</td>
<td>11%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Portfolio B dominates portfolio C because B has a higher expected return and a lower standard deviation. Thus, C is clearly inferior. A comparison of portfolios A and B represents a risk-return trade-off in that B has a higher expected return, but B also has higher risk. A crude comparison may use the coefficient of variation or the Sharpe measure, but a judgment regarding which portfolio is “better” would be based on the risk preference of the vice president.

Chapter 12: Loan Portfolio Risk – KMV Portfolio Manager

12. Country Side Bank uses the KMV Portfolio Manager model to evaluate the risk-return characteristics of the loans in its portfolio. A specific $10 million loan earns 2 percent per year in fees and the loan is priced at a 4 percent spread over the cost of funds for the bank. Because of collateral considerations, the loss to the bank if the borrower defaults will be 20 percent of the loan’s face value. The expected probability of default is 3 percent. What is the anticipated return on this loan? What is the risk of the loan?

\[
\text{Expected return} = \text{AIS}_i - \text{E}(L_i) = (0.02 + 0.04) - (0.03 \times 0.20) = .054 \text{ or } 5.4 \text{ percent}
\]

\[
\text{Risk of the loan} = \sigma_{D_i} \times \text{LGD}_i = [0.03(0.97)]^{0.5} \times 0.20 = 0.0341 \text{ or } 3.41 \text{ percent}
\]

13. Suppose that an FI holds two loans with the following characteristics.

<table>
<thead>
<tr>
<th>Annual</th>
<th>Spread Between Loan Rate and FI</th>
<th>Loss to FI</th>
<th>Expected Default</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
13. Suppose that an FI holds two loans with the following characteristics.

<table>
<thead>
<tr>
<th>Loan</th>
<th>X_i</th>
<th>Cost of Funds</th>
<th>Fees</th>
<th>Default</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.45</td>
<td>5.5%</td>
<td>2.25%</td>
<td>30%</td>
<td>3.5%</td>
</tr>
<tr>
<td>2</td>
<td>.55</td>
<td>3.5</td>
<td>1.75</td>
<td>20</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Calculate of the return and risk on the two-asset portfolio using KMV Portfolio Manager.

The return and risk on loan 1 are:

\[
R_1 = (.055 + .0225) - [.035 \cdot .30] = 0.0670 \text{ or } 6.70% \\
\sigma_1 = [\% .035(.965)] \cdot .30 = .05513 \text{ or } 5.513%
\]

The return and risk on loan 2 are:

\[
R_2 = (.035 + .0175) - [.01 \cdot .20] = 0.0505 \text{ or } 5.05% \\
\sigma_2 = [\% .01(.99)] \cdot .20 = .01990 \text{ or } 1.990%
\]

The return and risk of the portfolio is then:

\[
\begin{align*}
R_p &= .45 (6.70\%) + .55 (5.05\%) = 5.7925\% \\
\sigma_p &= (.45)^2 (5.513\%)^2 + (.55)^2 (1.990\%)^2 + 2 (.45) (.55)(-.15)(5.513\%)(1.990\%) = 6.53876%.
\end{align*}
\]

Thus, \(\sigma_p = 6.53876\% = 2.56\%\).

14. Suppose that an FI holds two loans with the following characteristics.

<table>
<thead>
<tr>
<th>Loan</th>
<th>X_i</th>
<th>Cost of Funds</th>
<th>Fees</th>
<th>Default</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>?</td>
<td>4.0%</td>
<td>1.50%</td>
<td>?%</td>
<td>4.0%</td>
</tr>
<tr>
<td>2</td>
<td>?</td>
<td>2.5</td>
<td>1.15</td>
<td>?</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The return on loan 1 is \(R_1 = 6.25\%\), the risk on loan 2 is \(\sigma_2 = 1.8233\%\), and the return of the portfolio is \(R_p = 4.555\%\). Calculate of the loss given default on loans 1 and 2, the proportions of loans 1 and 2 in the portfolio, and the risk of the portfolio, \(\sigma_p\), using KMV Portfolio Manager.
R_1 = .0625 = (.04 + .015) - [.040 \ H \ LGD_1] \Rightarrow LGD_1 = (.0625 - (.04 + .015))/(-.04) = .1875

\Rightarrow \sigma_1 = \[\% .04(.96)\] H .1875 = .03674 or 3.674%

\sigma_2 = .018233 = \[\% .015(.985)\] H LGD_2 \Rightarrow LGD_2 = .018233/\[\% .015(.985)\] = .15

\Rightarrow R_2 = (.025 + .0115) - [.015 H .15] = .03425 or 3.425%

\Rightarrow R_p = X_1 (.0625) + (1 - X_1) (.03425) = .04555

\Rightarrow X_1 = (.04555 - .03425)/(.0625 - .03425) = .40 and X_2 = 1 - .40 = .60

\sigma_p^2 = (.40)^2 (.03674)^2 + (.60)^2 (.018233)^2 + 2 (.40) (.60)(-.10)(.03674)(.018233) = .000303523.

Thus, \sigma_p = %.000303523 = .0174 = 1.74%.

15. What databases are available that contain loan information at national and regional levels? How can they be used to analyze credit concentration risk?

Two publicly available databases are (a) the commercial bank call reports of the Federal Reserve Board which contain various information supplied by banks quarterly and (b) the shared national credit database, which provides information on loan volumes of FIs separated by two-digit SIC (Standard Industrial Classification) codes. Such data can be used as a benchmark to determine whether an FI’s asset allocation is significantly different from the national or regional average.

Chapter 12: Loan Portfolio Risk – Loan Portfolio VaR

22. From Table 12A-1, what is the probability of a loan upgrade? A loan downgrade?

The probability of an upgrade is 5.95% + 0.33% + 0.02% = 6.30%. The probability of a downgrade is 5.30% + 1.17% + 0.12% = 6.59%.

a. What is the impact of a rating upgrade or downgrade?
The effect of a rating upgrade or downgrade will be reflected on the credit-risk spreads or premiums on loans and thus on the implied market value of the loan. A downgrade should cause this credit-risk spread premium to rise.

b. How is the discount rate determined after a credit event has occurred?

The discount rate for each year in the future in which cash flows are expected to be received includes the forward rates from the current Treasury yield curve plus the annual credit spreads for loans of a particular rating class for each year. These credit spreads are determined by observing the spreads of the corporate bond market over Treasury securities.

c. Why does the probability distribution of possible loan values have a negative skew?

The negative skew occurs because the probability distribution is non-normal. The potential downside change in a loan’s value is greater than the possible upside change in value.

d. How do the capital requirements of the CreditMetrics approach differ from those of the BIS and Federal Reserve System?

The Fed and the BIS require the capital reserve to be 8 percent of the risk-weighted book value of a loan. Under CreditMetrics each loan is likely to have a different VAR and thus a different implied capital requirement. Further, this required capital is likely to be greater than 8 percent of the risk-weighted book value because of the non-normality of the probability distributions.

23. A five-year fixed-rate loan of $100 million carries a 7 percent annual interest rate. The borrower is rated BB. Based on hypothetical historical data, the probability distribution given below has been determined for various ratings upgrades, downgrades, status quo, and default possibilities over the next year. Information also is presented reflecting the forward rates of the current Treasury yield curve and the annual credit spreads of the various maturities of BBB bonds over Treasuries.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Probability</th>
<th>New Loan Value plus Forward Rate Spreads at Time t</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Probability</td>
</tr>
<tr>
<td>AAA</td>
<td>0.01%</td>
<td>$114.82m</td>
</tr>
<tr>
<td>AA</td>
<td>0.31</td>
<td>114.60m</td>
</tr>
<tr>
<td>A</td>
<td>1.45</td>
<td>114.03m</td>
</tr>
<tr>
<td>BBB</td>
<td>6.05</td>
<td>108.55m</td>
</tr>
<tr>
<td>BB</td>
<td>85.48</td>
<td>108.55m</td>
</tr>
</tbody>
</table>
B 5.60  98.43m
CCC 0.90  86.82m
Default 0.20  54.12m

a. What is the present value of the loan at the end of the one-year risk horizon for the case where the borrower has been upgraded from BB to BBB?

\[
PV = \frac{7m}{1.0372} + \frac{7m}{(1.0436)^2} + \frac{7m}{(1.0491)^3} + \frac{107m}{(1.0530)^4} = 113.27 \text{ million}
\]

b. What is the mean (expected) value of the loan at the end of year one?

The solution table on the following page reveals a value of $108.06.

c. What is the volatility of the loan value at the end of the year?

The volatility or standard deviation of the loan value is $4.19.

d. Calculate the 5 percent and 1 percent VARs for this loan assuming a normal distribution of values.

The 5 percent VAR is 1.65 x $4.19 = $6.91.

The 1 percent VAR is 2.33 x $4.19 = $9.76.

| Year-end Value | Probability
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA 114.82</td>
<td>0.0001</td>
</tr>
<tr>
<td>AA 114.60</td>
<td>0.0031</td>
</tr>
<tr>
<td>A 114.03</td>
<td>0.0145</td>
</tr>
<tr>
<td>BBB 113.27</td>
<td>0.0605</td>
</tr>
<tr>
<td>BB 108.55</td>
<td>0.8548</td>
</tr>
<tr>
<td>B 98.43</td>
<td>0.056</td>
</tr>
<tr>
<td>CCC 86.82</td>
<td>0.009</td>
</tr>
<tr>
<td>Default 54.12</td>
<td>0.002</td>
</tr>
</tbody>
</table>

1.000 Mean = $108.06m  Variance = 17.5740
e. Estimate the “approximate” 5 percent and 1 percent VARs using the actual distribution of loan values and probabilities.

\[
5\% \text{ VAR} = 95\% \text{ of actual distribution} = $108.06m - $98.43m = $9.63m
\]

\[
1\% \text{ VAR} = 99\% \text{ of actual distribution} = $108.06m - $86.82m = $21.24m
\]

where: \(5\% \text{ VAR}\) is approximated by \(0.056 + 0.009 + 0.002 = 0.067\) or 6.7 percent, and
\(1\% \text{ VAR}\) is approximated by \(0.009 + 0.002 = 0.011\) or 1.1 percent.

Using linear interpolation, the \(5\% \text{ VAR} = $10.65\) million and the \(1\% \text{ VAR} = $19.31\) million. For the \(1\% \text{ VAR}\), \(\$19.31m = (1 - 0.1/1.1) \times \$21.24m\).

f. How do the capital requirements of the 1 percent VARs calculated in parts (d) and (e) above compare with the capital requirements of the BIS and Federal Reserve System?

The Fed and BIS systems would require 8 percent of the loan value, or \$8\) million. The 1 percent VAR would require \$19.31\) million under the approximate method, and \$9.76\) million (2.33 \times \$4.19m) in capital under the normal distribution assumption. In each case, the amounts exceed the Fed/BIS amount.

25. An FI has a loan portfolio of 10,000 loans of \$10,000 each. The loans have a historical average default rate of 4 percent and the severity of loss is 40 cents per dollar.

a. Over the next year, what are the probabilities of having default rates of 2, 3, 4, 5, and 8 percent?

\[
\text{Probability of 2 defaults} = \frac{e^{-m}m^n}{n!} = \frac{(2.71828)^{-4} \times 4^2}{1 \times 2} = \frac{0.018316 \times 16}{2} = 0.1465 = 14.65\%
\]

<table>
<thead>
<tr>
<th>n</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>14.65%</td>
<td>19.54%</td>
<td>19.54%</td>
<td>15.63%</td>
<td>2.98%</td>
</tr>
</tbody>
</table>

b. What would be the dollar loss on the portfolios with default rates of 4 and 8 percent?

Dollar loss of 4 loans defaulting = 4 \(\times 0.40 \times \$10,000 = \$16,000\)

Dollar loss of 8 loans defaulting = 8 \(\times 0.40 \times \$10,000 = \$32,000\)

c. How much capital would need to be reserved to meet the 1 percent worst-case loss scenario?

What proportion of the portfolio’s value would this capital reserve be?
The probability of 8 defaults is ~3 percent. The probability of 10 defaults is 0.00529 or close to 1 percent. The dollar loss of 10 loans defaulting is $40,000. Thus, a 1 percent chance of losing $40,000 exists.

A capital reserve should be held to meet the difference between the unexpected 1 percent loss rate and the expected loss rate of 4 defaults. This difference is $40,000 minus $16,000 or $24,000. This amount is 0.024 percent of the total portfolio.