Note on US Treasury Securities

This note is meant to provide selected background material on the US Treasury market such as treasury auctions, Bloomberg quote conventions, discount yields, creation of synthetic bonds through cash-flow replication, and convergence trades which we did not discuss but touched upon when we mentioned LTCM’s spectacular bankruptcy in 1998.

1. Treasury Auction Schedule

The reference points, i.e. “benchmarks,” for fixed-income pricing are the securities sold by the Treasury when they are on the run

- A Treasury is on the run until the next Treasury of that maturity becomes available
- On 11/15/07, the Treasury issued a ten-year note (i.e. maturing 11/15/17), which will be the on-the-run ten-year until 2/15/08, when it will issue a new ten-year note (maturing 2/15/18), which will be the on-the-run note until the Treasury issued the next ten-year on 5/15/08, at which point the ten-year maturing 11/15/17 will become the old ten-year.
- Sometimes the next auction is of the same security
- On 2/15/08 the Treasury will issue a thirty-year note, maturing 2/15/38, and on 5/15/08 the Treasury is planning to issue more of this same note (by then 29 ¾ years to maturity), called re-opening

Tentative Auction Schedule

Generally:

Bills:
- 13 & 26 Week Every Thursday (auction Monday)
- 4 Week Every Thursday (auction Tuesday) (re-opening)

Notes:
- 2 Year & 5-Year End of each month
- 10-Year 15th of Feb, May, Aug and Nov

Bonds:
- 30-year 15th of Feb (reopened in August)

Treasury also issues TIPS (Principal amount increases with CPI)

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1 This note is based on material by Professor David Musto, Wharton.
There are seven on-the-runs: 4 weeks, 3 months, 6 months, 2 years, 5 years, 10 years and the 30-year (sometimes 3-years as well)

- Almost all discussion about the Treasury market is about these securities
- “Treasury Yield Curve” in the newspaper plots only these points

Trading is concentrated in on-the-run issues

- Much higher volume, smaller spreads
  - 64% of trade volume is of on-the-run securities
  - 12% is of when-issued securities
    - Announced, but not auctioned yet
  - 24% is of all the off-the-run securities put together
- Higher prices (reflecting higher liquidity)
- More about this at the end of class

When Issued – Treasuries start trading **before** they even exist, in the when-issued market

- If you buy in this market, it is understood that you will get your security when it is issued
  - The seller then has to buy the necessary securities in the auction, or the subsequent secondary market, to deliver to you
  - Squeezing sellers in the when-issued market was the apparent goal in the Treasury-auction scandal in the early 90’s
    - Any one bidder can’t buy more than 35% of an issue
      - When-issued purchases plus auction purchases
      - Rule is intended to reduce fears of cornering the market
      - In the scandal, Salomon controlled ~92% of an issue
        - Trick was, placing bids in the names of customers who weren’t actually buying
          - This trick is not legal
  - The focus of trading shifts from the old bond to the new one when the new one starts trading on a when-issued basis

If we sort bonds into 1) on the run, 2) just off the run, and 3) off the run, we see this liquidity advantage

- Smaller spreads (Ask minus Bid)
- Larger **Quoted Depth** – amount the market maker will trade at his quoted spread
Spreads increase substantially as the security goes off the run.

Quoted depth shrinks substantially.
As an example consider the following quotes representing all the actives and recent notes on 1/2/08:

### U.S. Treasury Actives

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Price</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>0117700</td>
<td>3 1/8 10/19</td>
<td>3.217</td>
<td>10.580</td>
</tr>
<tr>
<td>01040038</td>
<td>3 2/3 3/30</td>
<td>3.425</td>
<td>10.800</td>
</tr>
<tr>
<td>01000008</td>
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<tr>
<td>0212008</td>
<td>103-21 1/4</td>
<td>103.264</td>
<td>103.275</td>
</tr>
<tr>
<td>0210009</td>
<td>101-30 1/4</td>
<td>103.000</td>
<td>103.01</td>
</tr>
<tr>
<td>0210010</td>
<td>100-21 1/8</td>
<td>100.142</td>
<td>100.142</td>
</tr>
<tr>
<td>0210011</td>
<td>99-23 1/2</td>
<td>100.342</td>
<td>100.342</td>
</tr>
<tr>
<td>0210012</td>
<td>99-24 5/8</td>
<td>100.161</td>
<td>100.161</td>
</tr>
<tr>
<td>0210013</td>
<td>98-26 3/4</td>
<td>100.000</td>
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</tr>
<tr>
<td>0210014</td>
<td>97-18 1/2</td>
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<tr>
<td>0210015</td>
<td>96-18 1/2</td>
<td>100.000</td>
<td>100.000</td>
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<tr>
<td>0210016</td>
<td>96-18 1/2</td>
<td>100.000</td>
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### 10-Year Notes

<table>
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<th>Description</th>
<th>Price</th>
<th>Yield</th>
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</thead>
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<tr>
<td>0210017</td>
<td>103-22 1/8</td>
<td>103.264</td>
<td>103.275</td>
</tr>
<tr>
<td>0210018</td>
<td>102-21 1/8</td>
<td>102.000</td>
<td>102.01</td>
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<tr>
<td>0210019</td>
<td>101-21 1/8</td>
<td>100.142</td>
<td>100.142</td>
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<tr>
<td>0210020</td>
<td>99-23 1/2</td>
<td>100.342</td>
<td>100.342</td>
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<tr>
<td>0210021</td>
<td>99-24 5/8</td>
<td>100.161</td>
<td>100.161</td>
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<td>0210022</td>
<td>99-24 5/8</td>
<td>100.161</td>
<td>100.161</td>
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<tr>
<td>0210023</td>
<td>98-26 3/4</td>
<td>100.000</td>
<td>100.000</td>
</tr>
<tr>
<td>0210024</td>
<td>97-18 1/2</td>
<td>100.000</td>
<td>100.000</td>
</tr>
<tr>
<td>0210025</td>
<td>96-18 1/2</td>
<td>100.000</td>
<td>100.000</td>
</tr>
<tr>
<td>0210026</td>
<td>96-18 1/2</td>
<td>100.000</td>
<td>100.000</td>
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### Option Quotes

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Bid Price</th>
<th>Ask Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>MZ</td>
<td>105-23+ / 105-23 3/8</td>
<td>4.024</td>
<td>4.024</td>
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<tr>
<td></td>
<td>105-23+ / 105-24</td>
<td>4.024</td>
<td>4.023</td>
</tr>
<tr>
<td></td>
<td>105-23+ / 105-24 a</td>
<td>4.024</td>
<td>4.022</td>
</tr>
<tr>
<td>JF</td>
<td>105-23+ / 105-24 a</td>
<td>4.024</td>
<td>4.022</td>
</tr>
<tr>
<td>SC</td>
<td>105-23+ / 105-24 a</td>
<td>4.024</td>
<td>4.022</td>
</tr>
<tr>
<td>WS</td>
<td>105-23+ / 105-24 a</td>
<td>4.024</td>
<td>4.021</td>
</tr>
<tr>
<td></td>
<td>105-23+ / 105-24 a</td>
<td>4.024</td>
<td>4.021</td>
</tr>
<tr>
<td>GX</td>
<td>105-23+ / 105-24 a</td>
<td>4.024</td>
<td>4.021</td>
</tr>
</tbody>
</table>

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For comparison, here are all the quotes for one of those notes (the old 10-year):
A couple of remarks about Bloomberg quotes and market conventions:

- US Treasuries pay interest on the 15\textsuperscript{th} of the month they are due and 6 months later up to maturity by convention because coupon payments occur semi-annually.
- Bloomberg puts a ‘+’ sign behind quotes to signify that you have to add 1/64 to the price; for instance, US Treasury Active # 17), the 4.5% 2/36 (formerly 30Y), is quote at 100-24+/100-25+ so that you would sell at 100 + 24.5/32 = 100 + 24/32 + 1/64 flat to and buy at 100 + 25.5/32 flat from the trader; hence
- you always have to add accrued interest to the Bloomberg quotes for UST’s which is straightforward because you know the (i) settlement and (ii) coupon dates, the latter by convention.

2. Yield-to-Maturity and (Bill) Discount Rate

As you know, the yield to maturity is defined as the rate \( y \) such that if you discount all future cash flows at \( y \), you get the current ask price, i.e., the fixed-income instruments internal rate of return (IRR). The one twist to the formula is that you count time in \( \text{half} \) years, and discount at \( y/2 \) per half year.
- Also, \# \text{half years to the next coupon is} \( x = \frac{\text{(# days to next coupon)/(# days from previous coupon to next coupon)}}{2} \), and \# \text{half years to the coupon} \( m \) \text{ coupons after that} is \( m+x \)

Discount Rate: A Treasury Bill pays face value at maturity, but no coupons before then
- Trades at a discount from face value
- The discount rate is the size of that discount, normalized to a 360-day year

Definition: \( d = 100 \frac{360/n}{(F-P)/F} \)
- \( d = \text{discount rate} \)
- \( n = \# \text{ days to maturity} \)
- \( F = \text{face value} \)
- \( P = \text{current price} \)

Rearranging to solve for the price in terms of the discount rate \( d \), we get

\[
P = F - \frac{d}{100} \frac{n}{360} F
\]

This is way prices are generally quoted for discount securities, not just Treasury bills but any instrument that simply pays one big lump sum in the near future (the biggest example is Commercial Paper, which we also briefly discussed).

Note that the discount rate/yield does not actually “discount” T-bills but is the “rebate” from face which implies the interest rate on a given bill. A lot of confusion arises from this fact because T-bills belong the category of “discount securities.” To get the yield in terms of a true discount rate we compute the bond-equivalent yield (BEY).
Consider the on-the-run 13-week bill on the screen above
- 91 days from settlement, 1/3/08, to maturity, 4/03/08
- Discount rate is 3.18% at the bid, 3.16% at the ask

So the price to buy $F = $1,000,000 face value of this bill is

\[ 1,000,000 - (3.16/100)(91/360)1,000,000 = 992,012 \]

Notice that the yield-to-maturity of the bill is 3.217, higher than 3.16
- In fact, yield is higher than discount rate for all of the bills
- You can’t directly compare yields and discounts, they’re different units

3. Bonds vs. STRIPS

STRIPS, or Separate Trading of Registered Interest and Principal Securities are the different payments bonds make, trading separately.
- Also called zeros, since they pay zero coupons

A STRIP simply pays you $100 on a specific future date, so its price is the price you pay today to get exactly $100 on a date in the future
- Treasury securities are really just bundles of STRIPs. This implies a very simple relation between their prices

An example makes this clear. Consider the 6% 8/15/09 note. On 1/2/08, the prices to buy and sell this note were

<table>
<thead>
<tr>
<th>Coupon</th>
<th>Maturity</th>
<th>Bid/Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>6%</td>
<td>8/15/09</td>
<td>104-19+/21+</td>
</tr>
</tbody>
</table>

Accrued interest as of the settlement date 1/3/08 = (141/184)(6/2) = 2.2989, implying invoice prices of (note the implied maturity date on Aug 15 and, hence, accrued interest!)

- 104+19.5/32+2.2989 = 106.9083 at the bid
- 104+21.5/32+2.2989 = 106.9708 at the ask
This note has four remaining payment dates: 2/15/08, 8/15/08 and 2/15/09 when it pays coupons of 3, and 8/15/09 when it pays the coupon of 3 and repays the principal of 100.

For each of these dates, there are STRIPs trading, each paying $100 on the stated date and nothing else. The quoted prices we find are

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Bid/Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/15/08</td>
<td>99-21/21</td>
</tr>
<tr>
<td>8/15/08</td>
<td>98-05+/-06</td>
</tr>
<tr>
<td>2/15/09</td>
<td>96-26/26+</td>
</tr>
<tr>
<td>8/15/09</td>
<td>95-08+/-09+</td>
</tr>
</tbody>
</table>

So, if we want to buy the same cash flows as the 6% 8/15/09 note, then we need to buy (at the ask)
- $3 of the 2/15/08 STRIP, costing \( (3/100)(99+21/32) = 2.9897 \)
- $3 of the 8/15/08 STRIP, costing \( (3/100)(98+6/32) = 2.9456 \)
- $3 of the 2/15/09 STRIP, costing \( (3/100)(96+26.5/32) = 2.9048 \)
- $103 of the 8/15/09 STRIP, costing \( (103/100)(95+9.5/32) = 98.1558 \)

For a total cost of \( 106.9959 \)

Can we make money by buying the STRIPs, reconstituting them into the bond and selling the bond? No, because the 106.9959 cost of the STRIPs exceeds the 106.9083 we would get from selling the bond.

Can we make money by buying the bond, stripping it and selling the pieces? To figure this out, we need to value the STRIPs at their bid prices, because that’s what we’d get for selling them. If we plug in the bid prices rather than the asks to the calculation above, we get \( 106.9628 \)
- So we could buy the bond for 106.9708 and sell its’ pieces for 106.9628, and that’s not profitable either

So the market keeps these prices closely in line. Not a surprise; it’s a pretty simple arbitrage proposition otherwise.
4. Relative prices of three bonds with the same maturity

There are three securities maturing on 8/15/15:

<table>
<thead>
<tr>
<th>Coupon Rate</th>
<th>Maturity</th>
<th>Bid/Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.625</td>
<td>08/15/15</td>
<td>145-12+/14+</td>
</tr>
<tr>
<td>4.25</td>
<td>08/15/15</td>
<td>103-13/16</td>
</tr>
<tr>
<td>0</td>
<td>08/15/15</td>
<td>74-17/20+</td>
</tr>
</tbody>
</table>

implying prices of

<table>
<thead>
<tr>
<th></th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.625</td>
<td>149.4616</td>
<td>149.5241</td>
</tr>
<tr>
<td>4.25</td>
<td>105.0346</td>
<td>105.1284</td>
</tr>
<tr>
<td>0</td>
<td>74.5313</td>
<td>74.6406</td>
</tr>
</tbody>
</table>

We can easily show: with any two of these bonds we can replicate the cash flows of another

Suppose you had bonds 1, 2 and 3, with coupon rates $C_1$, $C_2$, and $C_3$, with
- The same coupon payment dates
- The same principal repayment date (i.e. same maturity)
You want a portfolio of bonds 1 and 2 with the same cash flows on every date as bond 3.

Simple algebra problem: find the $x$ such that $xC_1 + (1-x)C_2 = C_3$. This way, buying face value $F$ of bond 3 is exactly the same as buying face value $xF$ of bond 1, and $(1-x)F$ of bond 2. Why?
- On each coupon date you get $xF(C_1/2) + (1-x)F(C_2/2) = FC_3/2$
- At maturity you get $xF + (1-x)F = F$

$x = (C_3-C_2)/(C_1-C_2)$ solves the equation

For example: Replicate $1M$ face value of the 4.25 ($C_3=4.25$) with a combination of the 10.625 ($C_1=10.625$) and the 0 ($C_2=0$).
- $x = (4.25-0)/(10.625-0) = 0.4$
- Buy $xF = 0.4($1M) = $400,000 face value of the 10.625% bond
  - This costs $(400,000)(149.5241/100) = $598,097$ (using the ask price)
- Buy $(1-x)F = $600,000 face value of the 0% bond
  - Costs $600,000(74.6406/100)=$447,844
- Total cost is $598,097 + $447,844 = $1,045,941
- Buying $1M of the 4.25% bond directly costs $1,051,284, and selling it brings in $1,050,346
- So if you buy the portfolio of the 10.625 and the STRIP, i.e. the synthetic bond or the replicating portfolio, and sell the 4.25%, then
  - you make $1,050,346 - $1,045,941 = $4407 per $1M face value when you put on the trade
  - All future cash flows offset
This is a very small and simple example of the kind of arbitrage bond traders look to put on:

- Not as riskless as the strip/reconstitution trade above, though, since you can’t simply convert the synthetic 4.25% into a real one; instead, you have to wait until the prices get back in line.

Notice that our formula does not restrict $x$ to be between 0 and 1.

- If $x < 0$ then you are buying a negative amount of bond 1, or in other words, selling it, and you will get the bid price, not the ask.
- If $x > 1$ then you are selling, not buying, bond 2 (since $1-x < 0$).

BTW, bond traders would not have bothered to add in accrued interest before comparing the prices of the actual 4.25 bond with the portfolio of the 10.625 and the 0, since the accrued interest is the same either way.

- Notice, though, that is true only if you’re replicating the coupons of one bond with the coupons of another coupon-paying bond.

5. **Spread between off- and on-the-run**

Off-the-runs sometimes trade at a spread over on-the-runs, reflecting their lower liquidity. But this spread goes away when the on-the-run goes off-the-run, when the next bond is issued.

- If you think liquidity is not a problem for you, this looks like an arbitrage.
  - Pocket the price difference by selling the on-the-run and buying the most recent off-the-run (often called the “old” bond).
  - Wait until both are off-the-run, when the spread should be gone, and gradually unwind the position.

Here’s what this looked like in 1993. On 2/15/93, the Treasury issued a new 30-year bond, maturing 2/23. At that point, the old bond was the one issued 11/15/92, maturing 11/22. To execute the trade you would have gone:

- Long the old bond, i.e. the 11/22 bond, and
- Short the new bond, i.e. the 2/23 bond

And hold the position until the next 30-year is issued on 8/15/93:
The upper graph shows the yields of the two bonds, and the lower graph shows the yield of the old bond minus the yield of the new bond. This is what should start out high and then go to zero by the time the next bond is issued. You can see it starts around 6bp, and is 0bp by 8/15/93.

This is one of the typical trades Long-Term Capital Management (LTCM), the hedge fund which went spectacularly bust in November 1998, put on, and is a very simple but representative example of the kind of trade they looked for:

- Driven by a temporary imbalance in liquidity demands
- Converges by a known date in the future

Here’s the same trade four years later, when LTCM was active

- Long the 11/26 bond and short the 2/27 bond
- Hold from 2/15/97 to 8/15/97, when the next 30-year is issued
Looks very similar. Here’s the next opportunity for the same trade

- Long the 2/27 bond and short the 8/27 bond
- Hold until 11/27, when the next 30-year is issued
Here, we see that the profitability of the trade has shrunk: the spread is smaller in the first place, and it converges less.

- It was at this time, the end of 1997, that LTCM decided it didn’t have the trading opportunities to make decent returns on its invested capital, so it started giving capital back to investors.
- Although LTCM implicitly acknowledge that the heydays of arbitrage were over they kept at it; they would miss the capital which they had returned to their investors very soon.
Here’s the old bond/new bond trade in the 30-year bond last year:

We can see a small but steady convergence.